Course Content:
Introduction to basic definitions and principles, fluids Statics and applications, basic equations of fluid flow, flow of incompressible fluids, flow past immersed bodies, transportation and metering of fluids, agitation and mixing of liquids. 45h (T); C

Course Description:
The course is designed to remind students to basic concepts, dimensions and units of measurement and general overview of fluid dynamics. Basic equations of fluid flow and pressure-volume relationship, Flow over a surface, Flow in a pipe, Newtonian fluids and Non-Newtonian Fluids, Gas-liquid flow, Flow of solid-liquid mixtures, Flow of gas-solid mixtures, Pipe, Fittings and Valves, Pumps, fans, blowers and compressors. Measurement of Flowing Fluids: nozzles, orifice, venture tube etc, Types of mixing, mixing mechanisms, Power consumption in stirred vessels, Flow patterns in stirred tanks, Rate and time for mixing, mixing equipment
Course Justification:

Movement of various forms of fluids is the hallmark of chemical processing industry where so many phenomena occur due to the nature of the fluid being transported, the channel of transportation and the forces behind the transportation. Bulk of the production related cost are incurred in this segment of chemical processing. This course provides the understanding needed in selecting, designing and arranging an economically viable fluid transport systems that minimizes cost and maximizes profit.

The objectives of this course as an integral part for the award of B.Eng. are:

- to teach chemical engineering students the basic behavior of fluids, principles of fluid transport and the associated equipments
- that after completing the class, students will be able to use momentum, and energy balances to analyse and/or design fluid systems of interest to chemical unit operations.

Course Requirements:

This is a compulsory course for all students studying Engineering. In view of this, students are expected to participate in all the course activities and have minimum of 75% attendance to be able to write the final examination.

Methods of grading:

<table>
<thead>
<tr>
<th>No</th>
<th>Item</th>
<th>Score %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Assignment/Quiz / Monthly Test</td>
<td>10</td>
</tr>
<tr>
<td>2.</td>
<td>Mid Semester Test</td>
<td>20</td>
</tr>
<tr>
<td>3.</td>
<td>Examination</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>100</td>
</tr>
</tbody>
</table>

Course Delivery Strategies:

The lecture will be delivered through face-to-face method, lecture guide (lecture note) will be provided during lectures. Students will be encouraged and required to read around the topics. The delivery strategies will also be supported by tutorial sessions.
LECTURE SCHEDULE

**Week 1: Introduction to Basic Definition and Principles of Fluid Mechanics**

**Objective:** Students will be able to explain the basic concept of fluid mechanics and dimensional analysis.

**Description:** The class will be focused on definitions, types of fluids, units of measurements, properties of fluids, dimensional analysis, techniques of dimensional analysis, and interpretation of dimensionless numbers.

**Week 2-3: Fluid Static and its Applications**

**Objective:** Students will gain the necessary knowledge of the types of forces that keep fluids at rest.

**Description:** Definition of fluid static, pressure in fluid, different types of measured pressures, head of fluids and measurement of fluid pressure.

Student will also be assessed on the topics covered so far through a short Monthly Quiz.

**Week 4-6: Fluid Dynamics**

**Objective:** Student will be able to understand the motion of fluid and the forces, which keep the body of fluid in motion.

**Description:** Nature of fluid flows, basic equations of fluid flow, friction in pipes selection of pipe sizes, unsteady state flows.

**Week 7-8: Pumping of Liquids**

**Objective:** Students will be able to understand the nature of devices involved in transportation of fluids.

**Description:** Pumps: types, heads, power requirement, operating characteristics.

**Week 9: Non-Newtonian Fluids**

**Objective:** Students will be able to understand the nature of fluids that exhibit rheological properties and nonlinear relationships between stress and strain rate.

**Description:** Types of non-newtonian fluid, flow characteristics.

**Week 10: Mid Semester Test**

**Week 11-12: Flow Measurement**

**Objective:** Students will understand the mechanisms of various fluid flow-measuring device.

**Description:** Flow measuring apparatus, unsteady state problems.

**Week 13 - 14: Liquid Mixing**

**Objective:** Students will understand the mechanisms of fluid mixing and various equipment involved.

**Description:** Types of mixing, mixing mechanisms, Power consumption in stirred vessels, Flow patterns in stirred tanks, Rate and time for mixing, mixing equipment.

**Week 15: Revision/ Tutorial Exercises**
Objective: Student will have opportunity to ask questions on all the topics covered in the course.

Description: A general overview of the course will be made. Students are expected to seek explanation on any difficult concept or topic treated during the course.

LIST OF BOOKS FOR FURTHER READING:

1.1 Definitions

Fluid mechanics is the study of the behaviour of fluids under the influence of forces and it can be studied under two broad topics: fluid statics and fluid dynamics. Fluid statics is the study of the forces that keep fluids in static equilibrium while fluid dynamics deals with the motion of fluids and the forces that keep them in motion. The subject of transport phenomena includes three closely related topics: fluid dynamics, heat transfer, and mass transfer. Fluid dynamics involves the transport of momentum, heat transfer deals with the transport of energy, and mass transfer is concerned with the transport of mass of various chemical species.

A fluid is a substance that undergoes continuous deformation when subjected to a shear stress. Liquids and gases are called fluids. An attempt to change the shape of a mass of fluid results in layers of fluid sliding over one another until a new shape is attained. During the change in shape, shear stresses exist, the magnitude of which depends upon the viscosity of the fluid and the rate of sliding, but when a final shape has been reached, and all shear stresses will disappear. A fluid in equilibrium is free from shear stresses.

1.2 Types of Fluids

(i) According to their physical and rheological properties:

Fluids may be classified as ideal and real or non-ideal fluids. The ideal fluids do not exhibit viscous properties and cannot sustain frictional and shear stresses when in motion. Its motion is being purely sustained by pressure forces and so, it cannot dissipate mechanical energy into heat. The real fluid possesses viscous properties, sustains frictional and shear stresses and dissipates mechanical energy into heat. In practice, the ideal fluid does not exist, but the flow of many real fluids can be analyzed by assuming that they are ideal especially if their viscosities are low.

(ii) According to their behaviour under the action of externally applied pressure.

Incompressible fluids are those whose volume of the element is independent of its pressure and temperature, but if its volume changes, it is said to be Compressible.

(iii) According to the effect produced by the action of a shear stress.

The most important physical properties affecting the stress distribution within the fluid is its viscosity. In many problem involving the flow of gas or liquid, the viscous stress are important and give rise to velocity gradients within the fluid, and dissipation of energy occurs as a result of the frictional forces set up. In gases and most pure liquids where the ratio of the shear stress to the rate of shear is constant and equal to the viscosity of the fluid, such fluids are said to be Newtonian in their behaviour. However, in some liquids, particularly those containing a second phase in suspension, the ratio is not constant and the apparent viscosity of the fluid is a function of
the rate of shear. The fluid is said to be **Non-Newtonian** and to exhibit rheological properties. Fluids that exhibit a nonlinear relationship between stress and strain rate are termed **non-Newtonian fluids**. Many common fluids that we see everyday are non-Newtonian. Paint, peanut butter, and toothpaste are good examples.

Consider two parallel plates of area \( A \), separated by distance \( dz \) apart shown in Figure 1.1. The space between the plates is filled with a fluid. The lower plate travels with a velocity \( u \) and the upper plate with a velocity \( u - du \). The small difference in velocity \( du \) between the plates results in a resisting force \( F \) acting over the plate area \( A \) due to viscous frictional effects in the fluid. Thus the force \( F \) must be applied to the lower plate to maintain the difference in velocity \( du \) between the two plates. The force per unit area \( F/A \) is known as the shear stress \( \tau \).

![Figure 1.1: Shear between two plates](image)

Newton’s law of viscosity states that:

\[
\tau \propto -\frac{du}{dz} \Rightarrow \tau = -\mu \frac{du}{dz}
\]

Fluids, which obey the equation 1.1, are called **Newtonian Fluids** while those that do not obey this equation, are called **non-Newtonian Fluids**. This law holds for Newtonian fluids in laminar flow. Momentum (shear stress) transfers through the fluid from the region of high velocities to region of low velocities, and the rate of momentum transfer increase with increasing the viscosity of fluids.

### 1.3 Dimensions and Units

#### 1.3.1 Dimensions

- mental concepts used to distinguish between opposing sense perceptions
- length (L), mass (m), time (t), temperature (T), amount of substance (n)
- fundamental dimensions - a base set from which all others can be derived

**Example.** What are the dimensions of mass flux (mass flow rate per unit area perpendicular to the flow)?

\[
G = \frac{1}{A} \frac{dm}{dt} \quad \text{dimensions are} \quad \frac{m}{L^2T}
\]

#### 1.3.2 Units

- scales used to quantify dimensions in a standard way
SI system is now almost universally accepted

the American engineering system (based on British standards) is still used extensively in the U.S.

1.3.3 Systems of Units

The system of units has the following components:

**Base units** are for mass, length, time, temperature, electrical current, and light intensity

**Multiple Units** are defined as multiples or fractions of base units such as minutes, hours etc.

**Derived units** are obtained by multiplying and dividing base or multiple units (cm², ft/min, kg-m/s², etc.).

(i) The base SI (Système Internationale d'Unités) units

- the ampere for electrical current and the candela for luminous
- the kelvin for temperature,
- meter (m) for length,
- the kilogram (kg) for mass,
- and the second (s) for time.

(ii) CGS system is almost identical to SI, the principal difference being that grams (g) and centimeters (cm) are used instead of kilograms and meters as the base units of mass and length.

(iii) American Engineering System

- foot (ft) for length,
- the pound-mass (lbm) for mass,
- and the second (s) for time.

For More Information check conversion factors on pages 1-4 through 1-20 of Perry’s Chemical Engineers’ Handbook.

**Example 1.1**: Convert 23lbm-ft/min² to its equivalent in kg·cm/s²

**Solution**

\[
\frac{23 \text{ lb}_m \cdot \text{ft}}{\text{min}^2} \times \frac{0.453593 \text{ kg}}{1 \text{ lb}_m} \times \frac{100 \text{ cm}}{3.281 \text{ ft}} \times \frac{1 \text{ min}^2}{(60 \text{ s})^2} = \frac{0.088 \text{ kg} \cdot \text{cm}}{\text{s}^2}
\]

**Example 1.2**

A supersonic aircraft consumes 5320 imperial gallons of kerosene per hour of flight and flies an average of 14 hours per day. It takes roughly seven tons of crude oil to produce one ton of kerosene. The density of kerosene is 0.965 g/cm³. How many planes would it take to consume the entire annual world production of 4.02 x 10⁹ metric tons of crude oil?

**Solution**
Table 1.1 SI and CGS Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit (SI)</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>meter (m)</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram (kg)</td>
<td>kg</td>
</tr>
<tr>
<td>Time</td>
<td>second (s)</td>
<td>s</td>
</tr>
<tr>
<td>Temperature</td>
<td>kelvin (K)</td>
<td>K</td>
</tr>
<tr>
<td>Electric Current</td>
<td>ampere (A)</td>
<td>A</td>
</tr>
<tr>
<td>Light Intensity</td>
<td>candela (cd)</td>
<td>cd</td>
</tr>
</tbody>
</table>

**Multiple Unit Prefixes**

- tera (T) = $10^{12}$
- giga (G) = $10^9$
- mega (M) = $10^6$
- kilo (k) = $10^3$
- centi (c) = $10^{-2}$
- milli (m) = $10^{-3}$
- micro (μ) = $10^{-6}$
- nano (n) = $10^{-9}$

Table 1.2 Derived Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Derived unit</th>
<th>Symbol</th>
<th>Relationship to primary units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force</td>
<td>newton (N)</td>
<td>N</td>
<td>kg m/s²</td>
</tr>
<tr>
<td>Work, energy, quantity of heat</td>
<td>joule (J)</td>
<td>J</td>
<td>N m</td>
</tr>
<tr>
<td>Power</td>
<td>watt (W)</td>
<td>W</td>
<td>J/s = N m/s</td>
</tr>
<tr>
<td>Area</td>
<td>square metre (m²)</td>
<td>m²</td>
<td></td>
</tr>
<tr>
<td>Volume</td>
<td>cubic metre (m³)</td>
<td>m³</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>kilogramme per cubic metre (kg/m³)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td>metre per second (m/s)</td>
<td>m/s</td>
<td></td>
</tr>
<tr>
<td>Acceleration</td>
<td>metre per second per second (m/s²)</td>
<td>m/s²</td>
<td></td>
</tr>
<tr>
<td>Pressure</td>
<td>pascal (Pa)</td>
<td>Pa</td>
<td>N/m²</td>
</tr>
<tr>
<td>Surface tension</td>
<td>newton per metre</td>
<td>N/m</td>
<td></td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>pascal second, or newton second per square metre (Pa s)</td>
<td>N m/s²</td>
<td></td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>square metre per second (m²/s)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1.3: Units Conversion Factor
# Unit Conversion Factors

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Equivalent Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1 kg = 1000 g = 0.001 metric ton (tonne) = 2.20461 lb, 35.27392 oz</td>
</tr>
<tr>
<td></td>
<td>1 lb = 453.593 g = 0.443593 kg = 5 x 10^-4 ton = 16 oz</td>
</tr>
<tr>
<td>Force</td>
<td>1 N = 1 kg m/s^2 = 10^5 dyn = 10^8 g cm/s^2 = 0.22418 lb</td>
</tr>
<tr>
<td></td>
<td>1 lb = 32.174 lb, ft/s^2 = 4.4482 N = 4.4482 x 10^5 dyn</td>
</tr>
<tr>
<td>Length</td>
<td>1 m = 100 cm = 10^8 µm = 10^10 Å = 39.37 in. = 3.2808 ft</td>
</tr>
<tr>
<td></td>
<td>1.0936 yd = 0.0006214 mi</td>
</tr>
<tr>
<td></td>
<td>1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm</td>
</tr>
<tr>
<td>Volume</td>
<td>1 m^3 = 1000 liters = 10^6 cm^3 = 35.3145 ft^3 = 264.17 gal</td>
</tr>
<tr>
<td></td>
<td>1 ft^3 = 1728 in.^3 = 7.4805 gal = 0.028317 m^3 = 28.317 liters</td>
</tr>
<tr>
<td></td>
<td>28,317 cm^3</td>
</tr>
<tr>
<td>Pressure</td>
<td>1 atm = 1.01325 x 10^5 N/m^2 (Pa) = 1.01325 x 10^5 dyn/cm^2 = 760 mm Hg @ 0°C (torr) = 10.333 m H_2O @ 4°C = 33.9 ft H_2O @ 4°C = 29.921 in. Hg @ 0°C = 14.696 lb/in.^2 (psi)</td>
</tr>
<tr>
<td>Energy</td>
<td>1 J = 1 N m = 10^7 erg = 10^7 dyn cm = 2.667 x 10^-7 kWh = 0.23901 cal = 0.7376 ft lb = 9.486 x 10^-4 Btu [550 ft lb/ (hp h)]</td>
</tr>
<tr>
<td>Power</td>
<td>1 W = 1 J/s = 0.23901 cal/s = 0.7376 ft lb/s = 9.486 x 10^-4 Btu/s = 1 x 10^-3 kW = 1.341 x 10^-3 hp</td>
</tr>
<tr>
<td>Flow Rate</td>
<td>1 m^3/s = 35.3145 ft^3/s = 264.17 gal/s = 1.585 x 10^4 gal/min = 10^6 cm^3/s</td>
</tr>
<tr>
<td></td>
<td>1 gpm = 6.309 x 10^-5 m^3/s = 2.228 x 10^-5 ft^3/s = 63.09 cm^3/s</td>
</tr>
</tbody>
</table>

Example: The factor to convert Pa to psi is 14.696 psi/(1.01325 x 10^5 Pa)

Some values of the gas constant: R = 8.314 x 10^5 kg m^2/(s^2 kg mol K)
R = 3.614 x 10^4 g cm^2/(s^2 g mol K)
R = 82.05 cm^3 atm/(g mol K)
R = 4.98 atm/(mol K) or Btu/(lb mol °R)
R = 1545 ft lb/(lb mol °R)
R = 10.73 ft^3 psi/(lb mol °R)
R = 0.730 ft^3 atm/(lb mol °R)
1.4 Dimensional Analysis

Many important engineering problems cannot be solved completely by theoretical or mathematical methods. Problems of this type are especially common in fluid flow, heat flow and diffusional operations. One method of resolving such problems is empirical experimentation. The empirical method of obtaining an equation relating the factors to their effects is dimensional analysis. A dimensional analysis cannot be made unless enough is known about the physics of the situation to decide what variables are important in the problem and what physical laws would be involved in a mathematical solution if one were possible.

Dimensional analysis is used to determine which groupings of the flow properties affect the performance of the model and prototype. This approach saves energy and cost, and tremendously increases the chance of producing a near perfect design of the prototype at first attempt.

The procedure for obtaining the dimensionless parameters is called Dimensional Analysis. Dimensional analysis helps to reduce the number of variables by grouping them into dimensionless parameters. It is then easier to determine the functional relationship between the parameters by experiment. Where there are many parameters, the relative influence of each parameter can be determined from experiment; less influential parameters can be dropped while the more influential parameters can be related empirically using appropriate coefficients and exponents. Many approaches have been proposed for converting a functional relationship in dimensional variables to a functional relationship in non-dimensional or dimensionless variables. Among them are, the step-by-step method, Buckingham II Theorem.

The dimensions of a quantity identify the physical character of that quantity, e.g., force (F), mass (M), length (L), time (t), temperature (T), electric charge (e), etc. On the other hand, Units identify the reference scale by which the magnitude of the respective physical quantity is measured. Many different reference scales (units) can be defined for a given dimension; for example, the dimension of length can be measured in units of miles, centimeters, inches, meters, yards, angstroms, furlongs, light years, kilometers, etc.

Dimensions can be classified as either fundamental or derived. Fundamental dimensions cannot be expressed in terms of other dimensions and include length (L), time (t), temperature (T), mass (M), and/or force (F) (depending upon the system of dimensions used). Derived dimensions can be expressed in terms of fundamental dimensions, for example, area ([A] = L²), volume ([V] =L³), energy ([E]=FL=ML²/t²), power ([HP]=FL/t=ML²/t³), viscosity ([μ]=Ft/L²=M/Lt) e.t.c.

1.4.1 Techniques of Dimensional Analysis
1.4.1.1 Step-by-Step Method

In this procedure, dimensions of variables of functional relationship are rendered dimensionless in mass, length and time by step by step. At each step, one of the variables is taken and combined with it to eliminate it and with others to render them dimensionless in whatever dimensions one desire. It may be necessary to use multiples of the variables in rendering some of the others dimensionless.

For example, the pressure drop \( \Delta p \) experienced by a fluid in turbulent motion over a length \( l \) of a smooth pipe has been found to be a function of the fluid velocity \( V \), the pipe diameter \( D \), the fluid density \( \rho \), and the fluid viscosity \( \mu \). Use the step-by-step method to reduce the functional relationship to that of dimensionless terms.

**Example 1.3**: The relationship can be expressed as follows

\[
\frac{\Delta p}{l} = f(V, D, \rho, \mu)
\]

The dimensions of the variables are:

\[
\begin{align*}
[\Delta p] &= \frac{M}{L^2 T^2} \\
[V] &= \frac{L}{T} \\
[D] &= L \\
[\rho] &= \frac{M}{L^3} \\
[\mu] &= \frac{M}{L T}
\end{align*}
\]

We can render the variables dimensionless in mass (M) by dividing them through by \( \rho \) to obtain

\[
\frac{\Delta p}{l \rho} = f \left( V, D, \frac{\rho}{\rho}, \frac{\mu}{\rho} \right)
\]

And since \( \rho/\rho = 1 \),

\[
\frac{\Delta p}{l \rho} = f \left( V, D, \frac{\mu}{\rho} \right)
\]

The dimensions of the terms in this equation are:

\[
\begin{align*}
[\Delta p/\rho] &= \frac{L}{T^2} \\
[V] &= \frac{L}{T} \\
[D] &= L \\
[\mu/\rho] &= \frac{L^2}{T}
\end{align*}
\]

Next, we render the equation dimensionless in length L by dividing \( \Delta p/\rho \), \( V \), and \( D \) by \( D \) and \( \mu/\rho \) by \( D^2 \) to obtain.
\[ \frac{\Delta p}{l \rho D} = f_2 \left( \frac{V}{D}, \frac{\mu}{\rho D^2} \right) \]

With the dimensions
\[
\begin{align*}
\frac{\Delta p}{l \rho D} & = \frac{1}{T^2} \\
\frac{V}{D} & = \frac{1}{T} \\
\frac{\mu}{\rho D^2} & = \frac{1}{T}
\end{align*}
\]

And finally, we render this equation dimensionless in time (T) by dividing the left term by \((\mu/\rho D^2)^2\) and the right hand terms by just \(\mu/\rho D^2\) to obtain
\[ \frac{\Delta ppD^3}{l \mu^2} = f_3 \left( \frac{\rho VD}{\mu} \right) \]

Inspection will show that both terms of this equation are dimensionless. Note that in the last step, we used \(\mu/\rho D^2\) to render the equation dimensionless in T; we could have used \(V/D\) and obtain
\[ \frac{\Delta pD^2}{l \rho DV^2} = f_4 \left( \frac{\mu}{\rho VD} \right) \]

Note that the final results are not the same, only as a result of the choice of the variable we have used in non-dimensionalising in T. Both approaches are correct and experiment can be employed to prove this point. In obtaining the dimensionless parameter in the above example, we adopted the order of M,L and T. Do note that there is no rigidity as to which order to adopt. We ended up with two parameter having carried out exercise on five variables employing three basic dimensions.

1.4.1.2 Buckingham Π Theorem

The theorem proves the conclusion we reached at the end of the last section that is, if a functional relationship contains \(m\) variables with a total of \(n\) basic dimensions, on the application of dimensional analysis, the relationship will contain \(m-n\) groups of dimensionless groups. In the step–by-step procedure, the dimensionless groups are determined together whereas in this procedure, the groups are determined one by one and each is designated as a Π.

The Buckingham’s Π-theorem is based on the following steps:

1. First of all, write the functional relationship with the given data.
2. Then write the equation in its general form.
3. Now choose \( m \) repeating variables (or recurring set) and write separate expressions for each \( \Pi \)-term. Every \( \Pi \)-term will contain the repeating variables and one of the remaining variables. Just the repeating variables are written in exponential form.

4. With help of the principle of dimensional homogeneity find out the values of powers \( a, b, c, \ldots \) by obtaining simultaneous equations.

5. Now substitute the values of these exponents in the \( \Pi \)-terms.

6. After the \( \Pi \)-terms are determined, write the functional relation in the required form.

**Note:**

Any \( \Pi \)-term may be replaced by any power of it, because the power of a non-dimensional term is also non-dimensional e.g. \( \Pi_1 \) may be replaced by \( \Pi_1^2, \Pi_1^3, \Pi_1^{0.5}, \ldots \) or by \( 2\Pi_1, 3\Pi_1, \Pi_1/2, \ldots \) etc.

**Selection of repeating variables**

In the previous section, we have mentioned that we should choose \( (m) \) repeating variables and write separate expressions for each \( \Pi \)-term. Though there is no hard or fast rule for the selection of repeating variables, yet the following points should be borne in mind while selecting the repeating variables:

1. The variables should be such that none of them is dimensionless.
2. No two variables should have the same dimensions.
3. Independent variables should, as far as possible, be selected as repeating variables.
4. Each of the fundamental dimensions must appear in at least one of the \( m \) variables.
5. It must not be possible to form a dimensionless group from some or all the variables within the repeating variables. If it were so possible, this dimensionless group would, of course, be one of the \( \Pi \)-term.
6. In general the selected repeating variables should be expressed as the following:
   
   (i) representing the flow characteristics,
   (ii), representing the geometry and
   (iii) representing the physical properties of fluid.

7. In case of that the example is held up, then one of the repeating variables should
   
   (i) representing the flow characteristics, (ii), representing the geometry and (iii) representing the physical properties of fluid.

7. In case that the example is held up, then one of the repeating variables should be changed.

The procedure is as follows as illustrated in the following example.

**Example 1.4:** Solve the problem \( \frac{\Delta p}{l} = f(V, D, \rho, \mu, g) \) using Buckingham’s procedure.

**Solution:**
The problem can be stated as
\[ f \left( \frac{\Delta p}{l}, V, D, \rho, \mu, g \right) = 0 \]

There are 6 variables and 3 basic dimensions M, L and T. Therefore, the solution should yield 3 \( \Pi \) terms. In selecting the repeating variables, it has been found to be helpful to the interpretation of the functional relationship to select one to reflect the rate of flow of the fluid, another to reflect mass of the fluid and the last one to reflect the characteristic dimension of the conduit. Consequently, for this problem, \( V, \rho \) and \( D \) are selected. The \( \Pi \) terms are therefore:
\[
\Pi_1 = V^{a_1} \rho^{a_2} D^{a_3} \frac{\Delta p}{l} \\
\Pi_2 = V^{b_1} \rho^{b_2} D^{b_3} \mu \\
\Pi_3 = V^{c_1} \rho^{c_2} D^{c_3} g
\]

Substituting dimensions for the variables, we have,
\[
M^0 L^0 T^0 = \left( \frac{L}{T} \right)^{a_1} \left( \frac{M}{L^3} \right)^{a_2} \left( \frac{M}{L^2 T^2} \right)^{a_3} \\
M^0 L^0 T^0 = \left( \frac{L}{T} \right)^{b_1} \left( \frac{M}{L^3} \right)^{b_2} \left( \frac{M}{L T^2} \right)^{b_3} \\
M^0 L^0 T^0 = \left( \frac{L}{T} \right)^{c_1} \left( \frac{M}{L^3} \right)^{c_2} \left( \frac{L}{T^2} \right)^{c_3}
\]

Equating the exponents for M, L and T in the first equation above gives:

M:
\[ a_2 + 1 = 0 \]
L:
\[ a_1 - 3a_2 + a_3 - 2 = 0 \]
T:
\[ -a_1 - 2 = 0 \]

Solving these three equations gives: \( a_1 = -2, a_2 = -1 \) and \( a_3 = 1 \)

Therefore,
\[ \Pi_1 = \frac{\Delta p}{l} \frac{D}{\rho V^2} \]

Equating the exponents for M, L and T in the second equation above gives:

M:
\[ b_2 + 1 = 0 \]
L:
\[ b_1 - 3b_2 + b_3 - 1 = 0 \]
T:
\[ -b_1 - 1 = 0 \]

Solving these three equations gives: \( b_1 = -1, b_2 = -1 \) and \( b_3 = -1 \)

Therefore
\[ \Pi_2 = \frac{\mu}{\rho V D} \]

Equating the exponents for \( M \), \( L \) and \( T \) in the third equation above gives:

- **M**: \( c_2 = 0 \)
- **L**: \( c_1 - 3c_2 + c_3 + 1 = 0 \)
- **T**: \(-c_1 - 2 = 0\)

Solving these three equations gives \( c_1 = 2; c_2 = 0 \) and \( c_3 = 1 \)

And consequently,

\[ \Pi_3 = \frac{gD}{V^2} \]

Finally, the resulting dimensionless equation is

\[ f_2 \left( \frac{\Delta p}{1}, \frac{D}{\rho V^2}, \frac{\mu}{\rho V D}, \frac{gD}{V^2} \right) = 0 \]

### 1.4.1.3 Rayleigh’s method (or Power series)

In this method, the functional relationship of some variable is expressed in the form of an exponential equation, which must be dimensionally homogeneous. If \( y \) is some function of independent variables \( (x_1, x_2, x_3 \ldots) \), then functional relationship may be written as;

\[ y = (x_1, x_2, x_3 \ldots) \]

The dependent variable \( y \) is one about which information is required; whereas the independent variables are those, which govern the variation of dependent variables.

The Rayleigh’s method is based on the following steps:-

1- First of all, write the functional relationship with the given data.
2- Now write the equation in terms of a constant with exponents i.e. powers \( a, b, c, \ldots \)
3- With the help of the principle of dimensional homogeneity, find out the values of \( a, b, c, \ldots \) by obtaining simultaneous equation and simplify it.
4- Now substitute the values of these exponents in the main equation, and simplify it.

**Example 1.5**

The thrust \( P \) of a propeller depends upon diameter \( D \); speed \( u \) through a fluid density \( \rho \); revolution per minute \( N \); and dynamic viscosity \( \mu \) Show that:

\[ P = (\rho D^2 u^3) f \left( \frac{\mu}{\rho D u}, \frac{D N}{u} \right), \text{ where } f \text{ is any function.} \]
1.4.2 Interpretation of Dimensionless Numbers

A lists some dimensionless groups that are commonly encountered in fluid mechanics problems are as listed in Table 1.4. The name of the group, and its symbol, definition, significance, and most common area of application are given in the table. Wherever feasible, it is desirable to express basic relations (either theoretical or empirical) in dimensionless form, with the variables being dimensionless groups, because this represents the most general way of presenting results and is independent of scale or specific system properties.

EXERCISES

1.1 (a) Convert 760 miles/h to m/s (b) Convert 921 kg/m\(^3\) to lbm/ft\(^3\) (c) 5.37 x 10\(^3\) kJ/min to hp.

1.2. Using only exact definitions and standards, calculate factors for converting
(a) newtons to pond force (b) British thermal units to IT calories (c) atmosphere to pond force per square inch (d) horsepower to kilowatts

1.3. Resistance force of a sphere immersed in a uniform flow (F\(_D\)) is a function of density (\(\rho\)), velocity (U), diameter of the sphere (D) and viscosity of the fluid (\(\mu\)). Get the dimensionless variables that govern the friction of a sphere in a uniform flow using Buckingham’s II Theorem.
Table 1.4 Dimensionless Groups in Fluid Mechanics

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Formula</th>
<th>Notation</th>
<th>Significance</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Archimedes number</td>
<td>$N_A$</td>
<td>$N_A = \frac{\rho g \Delta \rho d^2}{\mu^2}$</td>
<td>$\rho = \text{fluid density}$, $\Delta \rho = \text{solid density}$, $\mu = \text{fluid density}$</td>
<td>(Buoyant x inertial)/ viscous forces</td>
<td>Setting particles, fluidization</td>
</tr>
<tr>
<td>Bingham number</td>
<td>$N_B$</td>
<td>$N_B = \frac{\tau_0 D}{\mu \infty V}$</td>
<td>$\tau_0 = \text{yield stress}$, $\mu \infty = \text{limiting viscosity}$</td>
<td>(Yield/viscous) stresses</td>
<td>Flow of Bingham plastics</td>
</tr>
<tr>
<td>Bond number</td>
<td>$N_B$</td>
<td>$N_B = \frac{\Delta \rho g d^2}{\sigma}$</td>
<td>$\sigma = \text{surface tension}$</td>
<td>(Gravity/surface tension) forces</td>
<td>Rise or fall of drops or bubbles</td>
</tr>
<tr>
<td>Cauchy number</td>
<td>$N_C$</td>
<td>$N_C = \frac{\rho V^2}{K}$</td>
<td>$K = \text{bulk modulus}$</td>
<td>(Inertial/compressible) forces</td>
<td>Compressible flow</td>
</tr>
<tr>
<td>Euler number</td>
<td>$N_E$</td>
<td>$N_E = \frac{\Delta P}{\rho V^2}$</td>
<td>$\Delta P = \text{pressure drop in pipe}$</td>
<td>(Pressure energy)/ kinetic energy</td>
<td>Flow in closed conduits</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>$C_D$</td>
<td>$C_D = \frac{F_D}{\frac{1}{2} \rho V^2 A}$</td>
<td>$F_D = \text{drag force}$, $A = \text{area normal to flow}$</td>
<td>(Drag stress)/ (1/2 momentum flux)</td>
<td>External flows</td>
</tr>
</tbody>
</table>
| Fanning (Darcy) friction factor | $f$ or $f_o$ | \begin{align*} f &= \frac{\alpha D}{2 \nu^2 L} \\
\alpha &= \text{friction loss} \\
\nu &= \text{viscosity} \\
\beta &= \text{wall stress} \\
f &= \frac{\beta}{2 \nu^2} \end{align*} | (Energy dissipated)/ (KE of flow x 4L/D) or (Wall stress)/ (momentum flux) | Flow in pipes, channels, fittings, etc. |
| Froude number         | $N_F$  | $N_F = \frac{V}{g L}$ | $L = \text{characteristic length}$ | (Inertial/gravity) forces                   | Free surface flows                |
| Hedstrom number       | $N_H$  | $N_H = \frac{\tau_0 D^2 \rho}{\mu \infty}$ | $\tau_0 = \text{yield stress}$, $\mu \infty = \text{limiting viscosity}$ | (Yield x inertia)/ viscous stresses | Flow of Bingham plastics         |
| Reynolds number flows | $N_R$  | $N_R = \frac{DV \rho}{\mu}$ | $\rho = \text{mass density}$, $\mu = \text{kinematic viscosity}$ | (Inertial momentum flux)/ viscous momentum flux | Pipe/internal flows (Equivalent forms for external flows) |
| Mach number           | $N_M$  | $N_M = \frac{V}{c}$ | $c = \text{speed of sound}$ | (Gas velocity)/(speed of sound)             | High speed compressible flow      |

2.1 Definition

Fluid statics is the part of fluid mechanics that deals with fluids when there is no relative motion between the fluid particles. Typically, this includes two situations: when the fluid is at rest and when it moves like a rigid solid. This part will show how to calculate the pressure field in fluids at rest and how to calculate the interaction forces between the fluid and submerged surfaces.

The basic property of a static fluid is pressure. Pressure can be considered as a surface force exerted by a fluid against the walls of its container. For a static fluid, as shown by the following analysis, pressure turns out to be independent of the orientation of any internal surface on which the pressure is assumed to act.

Consider a mass of static fluid as shown in Figure 2.1. Imagine that the mass is isolated as a free body and consider all forces acting on it in the direction of the z-axis, either from outside the fluid or from the surrounding fluid. Three forces are involved: (i) the force of gravity acting downward (ii) the pressure force on plane $COB$ acting upward, and (iii) the vertical component of the pressure force on plane $ABC$ acting downward. Since the fluid is in equilibrium, the resultant of these forces is zero.

![Figure 2.1: Forces on static element of fluid](image)

2.2 Applications

2.2.1 Hydrostatic Equilibrium

In a stationary mass of a single static fluid, the pressure is constant in any cross section parallel to the earth's surface but varies from height to height. Consider the vertical column of fluid shown in Figure 2.2, where a stationary column of fluid of height ($h_2$) and cross-sectional area $A$, where $A = Ao = A_1 = A_2 = A_3$ as shown. The pressure above the fluid is $P$, it could be the pressure of atmosphere above the fluid. The fluid at any point, say $h_1$ must support all the fluid above it. It can be shown that the forces at any point in a static fluid must be the same in all directions. Also, for a fluid at rest, the pressure or (force / unit area) is the same at all points with the same elevation. For example, at $h_1$ from the top, the pressure is the same at all points on the cross-sectional area $A_1$. 
The total mass of fluid (kg) at height $h_2 = h_2A\rho$

The total force (N) of the fluid on area $A$ = $mg = h_2A\rho g$

The pressure (N/m$^2$ or Pa) : $P = F/A = h_2\rho g$

This is the pressure on $A_2$ due to the weight of the fluid column above it. However to get the total pressure $P_2$ on $A_2$, the pressure $P_o$ on the top of the fluid must be added:

$$P_2 = h_2\rho g + P_o$$

Thus to calculate $P_1$,

$$P_1 = h_1\rho g + P_o$$

The pressure difference between points $\odot$ and $\oplus$ is:

$$P_2 - P_1 = (h_2\rho g + P_o) - (h_1\rho g + P_o)$$

$$\Rightarrow P_2 - P_1 = (h_2 - h_1)\rho g$$  \text{SI units} \\
$$P_2 - P_1 = (h_2 - h_1)\rho g / g_c$$  \text{English units}

Since it is vertical height of a fluid that determines the pressure in a fluid, the shape of the vessel does not affect the pressure. For example in Figure 2.3 the pressure $P_1$ at the bottom of all three vessels is the same and equal to $(h_1\rho g + P_o)$

2.2.1.1 Definition of Absolute and Relative Pressure

Pressure is often associated with terms such as atmospheric, gauge, absolute, and vacuum. This section is dedicated to explaining their respective meanings.
(i) **Atmospheric Pressure**
It is the pressure exerted by atmospheric air on the earth due to its weight. This pressure changes as the density of air varies according to the altitudes. It may vary because of the temperature and humidity of air. Hence for all purposes of calculations, the pressure exerted by air at sea level is taken as standard and that is equal to:

\[
1 \text{ atm} = 1.01325 \text{ bar} = 101.325 \text{ kPa} = 10.328 \text{ m H}_2\text{o} = 760 \text{ torr (mm Hg)} = 14.7 \text{ psi}
\]

(ii) **Gauge Pressure or Positive Pressure**
It is the pressure recorded by an instrument. This is always above atmospheric. The zero mark of the dial will have been adjusted to atmospheric pressure.

(iii) **Vacuum Pressure or Negative Pressure**
This pressure is caused either artificially or by flow conditions. The pressure intensity will be less than the atmospheric pressure whenever vacuum is formed.

(iv) **Absolute Pressure**
Absolute pressure is the algebraic sum of atmospheric pressure and gauge pressure. Atmospheric pressure is usually considered as the datum line and all other pressures are recorded either above or below it.

\[
P_{\text{absolute}} = P_{\text{atmospheric}} + P_{\text{gauge}} = P_{\text{atmospheric}} - P_{\text{vacuum}}
\]

For example if the vacuum pressure is 0.3 atm => absolute pressure = 1.0 – 0.3 = 0.7 atm

2.2.1.2 *Head of Fluid*
Pressures are given in many different sets of units, such as N/m² or Pa, dyne/cm, psi, lbf/ft². However, a common method of expressing pressures is in terms of head (m, cm, mm, in, or ft) of a particular fluid. This height or head of the given fluid will exert the same pressure as the pressures it represents \( P = h \rho g \).
**Example 2.1**
A large storage tank contains oil having a density of 917 kg/m$^3$. The tank is 3.66 m tall and vented (open) to the atmosphere of 1 atm at the top. The tank is filled with oil to a depth of 3.05 m (10 ft) and also contains 0.61 m (2 ft) of water in the bottom of the tank. Calculate the pressure in Pa and psia at 3.05 m from the top of the tank and at the bottom. And calculate the gauge pressure at the bottom of the tank.

**Solution:**
\[ P_0 = 1 \text{ atm} = 14.696 \text{ psia} = 1.01325 \times 10^5 \text{ Pa} \]
\[ P_1 = h_1 \rho_{\text{oil}} g + P_o \]
\[ = 3.05 \text{ m} (917 \text{ kg/m}^3) 9.81 \text{ m/s}^2 + 1.01325 \times 10^5 \text{ Pa} \]
\[ = 1.28762 \times 10^5 \text{ Pa} \]
\[ P_1 = 1.28762 \times 10^5 \text{ Pa} (14.696 \text{ psia}/1.01325 \times 10^5 \text{ Pa}) \]
\[ = 18.675 \text{ psia} \]

or
\[ P_1 = h_1 \rho_{\text{oil}} g + P_o \]
\[ = 10 \text{ ft m} [917 \text{ kg/m}^3 (62.43 \text{ lb/ft}^3/1000 \text{ kg/m}^3)] (32.174 \text{ ft/s}^2/32.174 \text{ lb/ft/lb f.s}^2) \]
\[ 1/144 \text{ ft}^2/\text{in}^2 + 14.696 = 18.675 \text{ psia} \]

\[ P_2 = P_1 + h_2 \rho_{\text{water}} g \]
\[ = 1.28762 \times 10^5 \text{ Pa} + 0.61 \text{ m} (1000 \text{ kg/m}^3) 9.81 \text{ m/s}^2 \]
\[ = 1.347461 \times 10^5 \text{ Pa} \]
\[ P_2 = 1.347461 \times 10^5 \text{ Pa} (14.696 \text{ psia}/1.01325 \times 10^5 \text{ Pa}) \]
\[ = 19.5433 \text{ psia} \]

The gauge pressure = abs – atm
\[ = 33421.1 \text{ Pa} = 4.9472 \text{ psig} \]

**Example 2.2**
Convert the pressure of [1 atm = 101.325 kPa] to
a) head of water in (m) at 4°C
b) head of Hg in (m) at 0°C
Solution:
a- The density of water at 4°C is approximately 1000 kg/m³
  \[ h = \frac{P}{\rho_{\text{water}} g} = 1.01325 \times 10^3 \text{ Pa} (1000 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2) = 10.33 \text{ m H}_2\text{o} \]
b- The density of mercury at 0°C is approximately 13595.5 kg/m³
  \[ h = \frac{P}{\rho_{\text{mercury}} g} = 1.01325 \times 10^5 \text{ Pa} (13595.5 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2) = 0.76 \text{ m Hg} \]
or
  \[ P = (h \rho g)_{\text{water}} = (h \rho g)_{\text{mercury}} \Rightarrow h_{\text{Hg}} = h_{\text{water}} \left( \frac{\rho_{\text{water}}}{\rho_{\text{Hg}}} \right) \]
  \[ h_{\text{Hg}} = 10.33 \left( \frac{1000}{13595.5} \right) = 0.76 \text{ m Hg} \]

Example 2.3
Find the static head of a liquid of sp.gr. 0.8 and pressure equivalent to 5 x 10⁴ Pa.

Solution:
\[ \rho = 0.8 (1000) = 800 \text{ kg/m}^3 \]
\[ h = \frac{P}{\rho g} = 5 \times 10^4/(800 \times 9.81) = 6.37 \text{ m H}_2\text{o} \]

2.2.2 Measurement of Fluid Pressure

Various types of fluid pressure measuring devices are discussed in this section

1- Piezometer tube
The piezometer consists a tube open at one end to the atmosphere and the other end is being inserted into vessel or pipe of which pressure is to be measured. The height to which the liquid rises up in the vertical tube gives the pressure head directly. i.e. \( P = h \rho g \)
Piezometer is used for measuring moderate pressures. It is meant for measuring *gauge pressure only* as the end is open to atmosphere. It cannot be used for *vacuum pressures*.

![Piezometer](image)

Figure 2.4: The Piezometer

2- Manometers
The manometer is an improved (modified) form of a piezometer. It can be used for measurement of comparatively *high pressures* and of both *gauge and vacuum pressures*. Following are the various types of manometers:
- a- Simple manometer
- b- The well type manometer
- c- Inclined manometer
- d- The inverted manometer
- e- The two-liquid manometer
a Simple manometer

It consists of a transparent U-tube containing the fluid A of density \( \rho_A \) whose pressure is to be measured and an immiscible fluid (B) of higher density \( \rho_B \). The limbs are connected to the two points between which the pressure difference \( (P_2 - P_1) \) is required; the connecting leads should be completely full of fluid A. If \( P_2 \) is greater than \( P_1 \), the interface between the two liquids in limb 2 will be depressed a distance \( h_m \) (say) below that in limb 1.

The pressure at the level a — a must be the same in each of the limbs and, therefore:

\[
P_2 + Z_m \rho_A g = P_1 + (Z_m - h_m) \rho_A g + h_m \rho_B g
\]

\[
\Rightarrow \Delta p = P_2 - P_1 = h_m (\rho_B - \rho_A) g
\]

If fluid A is a gas, the density \( \rho_A \) will normally be small compared with the density of the manometer fluid \( \rho_m \) so that:

\[
\Delta p = P_2 - P_1 = h_m \rho_B g
\]

b The well-type manometer

In order to avoid the inconvenience of having to read two limbs, and in order to measure low pressures, where accuracy is of much importance, the well-type manometer shown in Figure (5) can be used. If \( A_w \) and \( A_c \) are the cross-sectional areas of the well and the column and \( h_m \) is the increase in the level of the column and \( h_w \) the decrease in the level of the well, then:

\[
P_2 = P_1 + (h_m + h_w) \rho g
\]

or:

\[
\Delta p = P_2 - P_1 = (h_m + h_w) \rho g
\]
The quantity of liquid expelled from the well is equal to the quantity pushed into the column so that:
\[ A_w h_w - A_c h_m = h_w - (A_c/A_w) h_m \]
\[ \Rightarrow \Delta p = P_2 - P_1 = \rho g h_m (1 + A_c/A_w) \]

If the well is large in comparison to the column then:
\[ i.e. (A_c/A_w) \rightarrow 0 \Rightarrow \Delta p = P_2 - P_1 = \rho g h_m \]

c- The inclined manometer
This is as shown in Figure 2.7 and it enables the sensitivity of the manometers described previously to be increased by measuring the length of the column of liquid. If \( \theta \) is the angle of inclination of the manometer (typically about 10 - 20°) and \( L \) is the movement of the column of liquid along the limb, then:
\[ h_m = L \sin \theta \]
If \( \theta = 10^\circ \), the manometer reading \( L \) is increased by about 5.7 times compared with the reading \( h_m \) which would have been obtained from a simple manometer.

d- The inverted manometer
Figure 2.8 is used for measuring pressure differences in liquids. The space above the liquid in the manometer is filled with air, which can be admitted or expelled through the tap A in order to adjust the level of the liquid in the manometer.
c- The two-liquid manometer

Small differences in pressure in gases are often measured with a manometer of the form shown in Figure 2.9. The reservoir at the top of each limb is of a sufficiently large cross-section for the liquid level to remain approximately the on each side of the manometer. The difference in pressure is then given by:

$$\Delta p = p_2 - p_1 = h \rho (\rho_{m1} - \rho_{m2}) g$$

where $\rho_{m1}$ and $\rho_{m2}$ are the densities of the two manometer liquids. The sensitivity of the instrument is very high if the densities of the two liquids are nearly the same. To obtain accurate readings it is necessary to choose liquids, which give sharp interfaces: paraffin oil and industrial alcohol are commonly used.

3- Mechanical Gauges

Whenever a very high fluid pressure is to be measured, and a very great sensitivity a mechanical gauge is best suited for these purposes. They are also designed to read vacuum pressure. A mechanical gauge is also used for measurement of pressure in boilers or other pipes, where tube manometer cannot be conveniently used. There are many types of gauge available in the market. But the principle on which all these gauge work is almost the same. The followings are some of the important types of mechanical gauges:

1- The Bourdon gauge
2- Diaphragm pressure gauge
3- Dead weight pressure gauge

The Bourdon gauge

The pressure to be measured is applied to a curved tube, oval in cross-section, and the deflection of the end of the tube is communicated through a system of levers to a recording needle. This gauge is widely used for steam and compressed gases, and frequently forms the indicating element on flow...
controllers. The simple form of the gauge is illustrated in Figures 2.9 a and b. Figure 2.9 c shows a Bourdon type gauge with the sensing element in the form of a helix; this instrument has a very much greater sensitivity and is suitable for very high pressures. It may be noted that the pressure measuring devices of category (2) all measure a pressure difference ($\Delta p = P_2 - P_1$).

In the case of the Bourdon gauge (1) of category (3), the pressure indicated is the difference between that communicated by the system to the tube and the external (ambient) pressure, and this is usually referred to as the **gauge pressure**. It is then necessary to add on the ambient pressure in order to obtain the **(absolute) pressure**.

Gauge pressures are not, however, used in the SI System of units.

**Example 2.4**

A simple manometer is used to measure the pressure of oil sp.gr. 0.8 flowing in a pipeline. Its right limb is open to atmosphere and the left limb is connected to the pipe. The center of the pipe is 9.0 cm below the level of the mercury in the right limb. If the difference of the mercury level in the two limbs is 15 cm, determine the absolute and the gauge pressures of the oil in the pipe.

**Solution**

**Given:**
- Density of the fluid ($\rho$) = 1000 kg/m$^3$ (0.8) = 800 kg/m$^3$
- Density of mercury ($\rho_m$) = 13600 kg/m$^3$
- Atmospheric pressure ($P_o$) = 1.01325 x 10$^5$ Pa
- Acceleration due to gravity (g) = 9.81 m/s$^2$

From the diagram:

$$P_1 = P_a + (800)(0.15-0.09)(9.81)$$
$$P_2 = P_o + (13600)(0.15)(9.81)$$

Thus:

$$P_a = P_o + (13600)(0.15)(9.81) - (800)(0.15-0.09)(9.81)$$
$$= 1.01325 \times 10^5 + 2.00124 \times 10^4 - 470.88$$
$$= 1.20866 \times 10^5 \text{ Pa}$$

**Therefore the Absolute Pressure ($P_a$) = 1.20866 x10$^5$ Pa**

The gauge pressure = Absolute Pressure - Atmos. Pressure

$$= 1.20866 \times 10^5 - 1.01325 \times 10^5$$

**The gauge pressure = 1.9541 x 10^4 Pa**
Example - 2.5
The following Figure shows a manometer connected to the pipeline containing oil of sp.gr. 0.8. Determine the absolute pressure of the oil in the pipe, and the gauge pressure. (Note that the manometer liquid is mercury with a density of 13600 kg/m$^3$)

Solution
\[ \rho_a = 1000 \text{ kg/m}^3 \times (0.8) = 800 \text{ kg/m}^3 \]
\[ P_1 = P_2 \]
\[ P_1 = P_o - h_2 \rho_a g \]
\[ P_2 = P_o + h_1 \rho_m g \]
\[ \Rightarrow P_a = P_o + h_1 \rho_m g + h_2 \rho_a g \]
\[ = 1.01325 \times 10^5 + (0.25) \text{ m} \]
\[ (13600 \text{ kg/m}^3) \times 9.81 \text{ m/s}^2 + (0.75) \text{ m} \times (800 \text{ kg/m}^3) \times 9.81 \text{ m/s}^2 \]
\[ = 1.40565 \times 10^5 \text{ Pa} \]

Example - 2.6
A conical vessel is connected to a U-tube having mercury and water as shown in the Figure. When the vessel is empty the manometer reads 0.25 m. find the reading in manometer, when the vessel is full of water.

Solution:
\[ P_1 = P_2 \]
\[ P_1 = (0.25 + H) \rho_w g + P_o \]
\[ P_2 = 0.25 \rho_m g + P_o \]
\[ \Rightarrow (0.25 + H) \rho_w g + P_o = 0.25 \rho_m g + P_o \]
\[ \Rightarrow H = 0.25 (\rho_m - \rho_w) / \rho_w \]
\[ = 0.25 (12600 /1000) = 3.15 \text{ m} \]
When the vessel is full of water, let the mercury level in the left limb go down by (x) meter and the mercury level in the right limb go up by the same amount (x) meter.

i.e. the reading manometer = (0.25 + 2x)
\[ P_1 = P_2 \]
\[ P_1 = (0.25 + x + H + 3.5) \rho_w g + P_o \]
\[ P_2 = (0.25 + 2x) \rho_m g + P_o \]
\[ \Rightarrow (0.25 + x + H + 3.5) \rho_w g + P_o = (0.25 + 2x) \rho_m g + P_o \]
\[ \Rightarrow 6.9 + x = (0.25 + 2x) (\rho_m / \rho_w) \Rightarrow x = 0.1431 \text{ m} \]
The manometer reading = 0.25 + 2 (0.1431) = 0.536 m
Example -2.7-
The following Figure shows a compound manometer connected to the pipeline containing oil of sp.gr. 0.8. Calculate $P_a$.

**Solution:**

\[
\rho_a = 0.8 \times 1000 = 800 \text{ kg/m}^3
\]

\[
P_a + 0.4 \rho_a g - 0.3 \rho_m g + 0.3 \rho_a g - 0.3 \rho_m g - P_o = 0
\]

\[
\Rightarrow P_a = P_o + 0.7 \rho_a g - 0.6 \rho_m g
\]

\[
= 1.01325 \times 10^5 - 0.7 \times (800) 9.81 + 0.6 \times (13600) 9.81
\]

\[
= 1.75881 \times 10^5 \text{ Pa}
\]

Example -2.8-
A differential manometer is connected to two pipes as shown in the Figure. The pipe A is containing $\rho_a$ carbon tetrachloride sp.gr. = 1.594 and the pipe B is contain an oil of sp.gr. = 0.8. Find the difference of mercury level if the pressure difference in the two pipes be 0.8 kg/cm².

**Solution:**

\[
P_1 = P_2
\]

\[
P_1 = P_B + (1 + h) \rho_b g
\]

\[
P_2 = P_A + 3.5 \rho_a g + h \rho_m g
\]

\[
\Rightarrow P_A - P_B = 3.5 \rho_a g + h \rho_m g - (1 + h) \rho_b g = (0.8 \text{ kg/cm}^2) (9.81 \text{ m/s}^2) (10^4 \text{ cm}^2/m^2)
\]

\[
\Rightarrow 7.848 \times 10^4 = 3.5 (1594) 9.81 + h (13600) 9.81 - (1+h) 800 (9.81)
\]

\[
\Rightarrow h = 25.16 \text{ cm}
\]
Exercise

2.1 Two pipes A and B carrying water are connected by a connecting tube as shown in Figure,

a- If the manometric liquid is oil of sp.gr. = 0.8, find the difference in pressure intensity at A and B when the difference in level between the two pipes be (h = 2 m) and (x = 40 cm).

b- If mercury is used instead of water in the pipes A and B and the oil used in the manometer has sp.gr. = 1.5, find the difference in pressure intensity at A and B when (h = 50 cm) and (x = 100 cm).

2.2 A closed vessel is divided into two compartments. These compartments contain oil and water as shown in Figure. Determine the value of (h).

2.3 Oil of sp.gr. = 0.9 flows through a vertical pipe (upwards). Two points A and B one above the other 40 cm apart in a pipe are connected by a U-tube carrying mercury. If the difference of pressure between A and B is 0.2 kg/cm² (a) Find the reading of the manometer.

(b) If the oil flows through a horizontal pipe, find the reading in manometer for the same difference in pressure between A and B.

2.4 A mercury U-tube manometer is used to measure the pressure drop across an orifice in pipe. If the liquid that flowing through the orifice is brine of sp.gr. 1.26 and upstream pressure is 2 psig and the downstream pressure is (10 mm Hg) vacuum, find the reading of manometer.

2.5 Three pipes A, B, and C at the same level connected by a multiple differential manometer shows the readings as show in Figure. Find the differential of pressure heads in terms of water column between A and B, between A and C, and between B and C.
3.1 Introduction

Movement of fluid from one unit operation to the other is a common phenomenon in the process industries. This movement may be over a long distance where substantial drop in pressure in both the pipeline and in individual units occurs as a result of interaction of different types of forces characterising the motion. It is necessary, therefore, to consider the problems associated with designing the most suitable flow system; estimating the most economical sizes of pipes; measuring the rate of flow, and frequently with controlling this flow at steady state. When a fluid is flowing over a surface or through a pipe, the velocity at various points in a plane at right angles to the stream velocity is rarely uniform, and the rate of change of velocity with distance from the surface will exert a vital influence on the resistance to flow and the rate of mass or heat transfer.

A scientist called Reynolds in 1883 observed that when a fluid flows through a tube or over a surface, the pattern of flow varies with the velocity, the physical properties of fluid, and the geometry of the surface. When the velocity of the fluid is slow, the flow pattern is smooth (laminar flow) and when it is high, an unstable pattern is observed in which eddies or small packets of fluid particles are present moving in all directions and at all angles to the normal line of flow (turbulent flow). He put forward a dimensionless number called Reynolds Number (Re) which combines the average fluid velocity \( u \), density \( \rho \), dynamic viscosity \( \mu \) and tube diameter \( d \) to characterise different types of flow. The Reynolds Number is as defined in equation (3.1)

\[
Re = \frac{\rho ud}{\mu} \tag{3.1}
\]

Also, given a volumetric flow rate \( Q (\text{m}^3/\text{s}) \), mass flow rate \( \dot{m} (\text{kg/s}) \), mass flux or mass velocity \( G \) (kg/m^2.s) and cross sectional area \( A \), then:

\[
u = \frac{Q}{A} = \frac{Q}{(\pi / 4)d^2} \tag{3.2}
\]

Then,

\[
Re = \frac{4Q\rho}{\dot{m}\mu} = \frac{4\dot{m}}{\dot{m}\mu} = \frac{Gd}{\mu} \tag{3.3}
\]

For a straight circular tube, when the value of \( Re < 2000 \), the flow is said to be laminar, \( Re > 4000 \) means turbulent flow and values in between means transition region.

Example 3.1

Water at 303 K is flowing at the rate of 10 gal/min in a pipe having an inside diameter I.D. of 2.067 in. Calculate the Reynolds number using both English and S.I. units.
3.2 Continuity Equation

The general principle of the conservation of mass in a system can be stated as shown in equation (3.4).

\[
\text{INPUT} = \text{OUTPUT} + \text{GENERATION} + \text{ACCUMULATION}
\]

However, in an event where the system is not reactive and operates at a steady state, the third and fourth terms are zero and thus results to equation (3.5)

\[
\text{INPUT} = \text{OUTPUT}
\]

Consider a simple flow system is shown in Figure 3.1 where fluid enters section 1 with an average velocity \((u_1)\) and density \((\rho_1)\) through a cross-sectional area \((A_1)\). The fluid leaves section 2 with an average velocity \((u_2)\) and density \((\rho_2)\) through the cross-sectional area \((A_2)\)

Thus at steady state:

\[
\dot{m}_1 = \dot{m}_2
\]
\[
Q_1 \rho_1 = Q_2 \rho_2
\]
\[
u_1 A_1 \rho_1 = u_1 A_1 \rho_2
\]

For incompressible fluids at the same temperature, \(\rho_1 = \rho_2\) and therefore

\[
u_1 A_1 = u_1 A_2
\]

Example 3.2
A petroleum crude oil having a density of 892 kg/m$^3$ is flowing, through the piping arrangement shown in the Figure below, at total rate of $1.388 \times 10^{-3}$ m/s entering pipe 1 through to pipe 2. The flow divides equally in each of pipes 3. The steel pipes are schedule 40 pipe. (See Table 3.1). Calculate the following using SI units:

a) The total mass flow rate in pipe 1 and pipes 3.
b) The average velocity in pipe 1 and pipes 3.
c) The mass velocity in pipe 1.

Solution

From Table 3.1 for schedule 40 pipe:

- 2" pipe has ID = 2.067"
- 3" has ID = 3.068"
- 1½" has ID = 1.610"

(Conversion 12" = 30.48 cm)

a- the total mass flow rate is the same through pipes 1 and 2 and is

$$\dot{m}_1 = Q_1 \rho = 1.388 \times 10^{-3} \text{ m}^3/\text{s} (892 \text{ kg/m}^3) = 1.238 \text{ kg/s}$$

Since the flow divides equally in each pipes 3,

$$\Rightarrow \dot{m}_3 = \dot{m}_1 / 2 = 1.238 / 2 = 0.619 \text{ kg/s}$$

b- $$\dot{m}_i = Q_i \rho = u_i A_i \rho \Rightarrow u_i = \frac{\dot{m}_i}{A_i \rho} = \frac{1.238 \text{ kg/s}}{(21.65 \times 10^{-4} \text{ m}^2)(892 \text{ kg/m}^3)} = 0.641 \text{ m/s}$$

$$u_3 = \frac{\dot{m}_3}{A_3 \rho} = \frac{0.619 \text{ kg/s}}{(13.13 \times 10^{-4} \text{ m}^2)(892 \text{ kg/m}^3)} = 0.528 \text{ m/s}$$

d- $$G_i = u_i \rho_1 = 0.641 \text{ m/s} (892 \text{ kg/m}^3) = 572 \text{ kg/m}^2 \cdot \text{s}$$

e- or $$G_i = \frac{\dot{m}_i}{A_i} = \frac{1.238 \text{ kg/s}}{21.65 \times 10^{-4} \text{ m}^2} = 572 \text{ kg/m}^2 \cdot \text{s}$$

3.3 Energy Relationships and Bernoulli’s Equation

The total energy of a fluid in motion consists of the following components:

(i) **Internal Energy (U)**: This is the energy associated with the physical state of fluid, i.e. the energy of atoms and molecules resulting from their motion and configuration. Internal energy is a function of temperature. It can be written as (U) energy per unit mass of fluid.

(ii) **Potential Energy (PE)**: This is the energy that a fluid has because of its position in the earth’s field of gravity. The work required to raise a unit mass of fluid to a height (z) above a datum line is (zag), where (g) is gravitational acceleration. This work is equal to the potential energy per unit mass of fluid above the datum line.

Table 3.2: Dimensions, Capacities and Weights of Standard Steel Pipes
Kinetic Energy (KE): This is the energy associated with the physical state of fluid motion. The kinetic energy of unit mass of the fluid is \((u^2/2)\), where \(u\) is the linear velocity of the fluid relative to some fixed body.
(iv) **Pressure Energy (Prss.E)**: This is the energy or work required to introduce the fluid into the system without a change in volume. If \(P\) is the pressure and \(V\) is the volume of a mass \(m\) of fluid, then \(\frac{PV}{m} = P_0\) is the pressure energy per unit mass of fluid. The ratio \(\frac{V}{m}\) is the fluid density \(\rho\).

Therefore, the total energy per unit mass is given by equation (3.7)

\[
E = U + zg + \frac{P}{\rho} + \frac{u^2}{2}
\]

The principle of the conservation of energy can be applied to a process of input and output streams for ideal fluid of constant density and without any pump present and no change in temperature.

\[
E_1 = E_2
\]

\[
U_1 + z_1g + \frac{P_1}{\rho} + \frac{u_1^2}{2} = U_2 + z_2g + \frac{P_2}{\rho} + \frac{u_2^2}{2}
\]

Then,

\[
z_1g + \frac{P_1}{\rho} + \frac{u_1^2}{2} = z_2g + \frac{P_2}{\rho} + \frac{u_2^2}{2}
\]

\[
\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + \Delta zg = 0 \quad \text{(Bernoulli's Equation)}
\]

### 3.4 Equation of Motion

In the fluid flow, the following forces are present:

1. \(F_g\): force due to gravity
2. \(F_p\): force due to pressure
3. \(F_v\): force due to viscosity
4. \(F_t\): force due to turbulence
5. \(F_c\): force due to compressibility
6. \(F_s\): force due to surface tension

The net force is could be given by equation 3.10 in x-direction

\[
F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x + (F_s)_x
\]

But for most problems in fluid motion, \((F_c)\) force due to compressibility and \((F_s)\) force due to surface tension are neglected and so equation (3.10) becomes equation (3.11) called Reynolds equation of motion which is useful in the analysis of turbulent flow.

\[
F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x
\]
In laminar (viscous) flow, the turbulent force becomes insignificant and hence the equation (3.11) may be written as (3.12) called Naiver-Stoke equation of motion useful for analysis of viscous flow:

\[ F_x = (F_g)_x + (F_P)_x + (F_V)_x \]  

3.12

If the flowing fluid is ideal and has very small viscosity, the viscous force and viscosity being almost insignificant and the equation (3.12) becomes (3.13) called Euler's equation of motion:

\[ F_x = (F_g)_x + (F_P)_x \]  

3.13

The pictorial representation of the equations is as given in Figure 3.2

**3.4.1 Euler's Equation of Motion**

Consider a steady flow of an ideal fluid along a streamline as shown in Figure 3.1. Now consider a small element of the flowing fluid:

Let:  
- dA: cross-sectional area of the fluid element,  
- dL: Length of the fluid element  
- dW: Weight of the fluid element  
- u: Velocity of the fluid element  
- P: Pressure of the fluid element

The Euler’s equation of motion assumptions are:

(i) The fluid is non-viscous (the frictional losses are zero).
(ii) The fluid is homogenous and incompressible (the density of fluid is constant).
(iii) The flow is continuous, steady, and along the streamline (laminar).
(iv) The velocity of flow is uniform over the section.
(v) No energy or force except gravity and pressure forces is involved in the flow.

The forces on the cylindrical elements are :

(i) Pressure force acting on the direction of flow (PdA)

(ii) Pressure force acting on the opposite direction of flow [(P+dP)dA]

(iii) Component of gravity force acting on the opposite direction of flow (dWsin θ)

- The Pressure force in the direction of flow :  
  \[ F_p = PdA - (P+dP)dA = -dPdA \]
- The gravity force in the direction of flow :  
  \[ F_g = -dW \sin \theta \]  
  \[ (W= mg = \rho dAdLg) \]
  \[ = -\rho gdAdLs\sin \theta \]  
  \[ (s\sin \theta = dz/dL) \]
  \[ F_g = -\rho gdAdz \]

- The net force in the direction of flow :  
  \[ F = ma \]
  \[ = \rho dAdL \]
  \[ a = \frac{du}{dt} = \frac{du}{dL} \times \frac{dL}{dt} = \frac{du}{dL} \]

Then ,

\[ F_x = (F_g)_x + (F_p)_x \]
\[ \rho dAdu = -dPdA - \rho gdAdz \]

Divide through by ρdA and rearranging gives
\[ \frac{dP}{\rho} + u_1 u_2 + d \frac{g}{z} = 0 \quad \text{(Euler's Equation of motion)} \]  \[3.14\]

Bernoulli’s equation could be obtained by integrating the Euler’s equation

\[ \int \frac{dP}{\rho} + \int u_1 u_2 + \int d \frac{g}{z} = 0 \]

\[ \frac{P}{\rho} + \frac{u_1^2}{2} + zg = \text{cons} \tan t \]

\[ \frac{\Delta P}{\rho} + \frac{\Delta u_1^2}{2} + \Delta zg = 0 \quad \text{(Bernoulli's Equation)} \]

### 3.4.2 Modifications to Bernoulli’s Equation

#### (i) Correction of the Kinetic Energy Term

The velocity in kinetic energy term is the mean linear velocity in the pipe. To account the effect of the velocity distribution across the pipe [(\(\alpha\)) dimensionless correction factor] is used.

For a circular cross sectional pipe:
- \(\alpha = 0.5\) for laminar flow
- \(\alpha = 1.0\) for turbulent flow

#### (ii) Modification for real fluid

The real fluids are viscous and hence offer resistance to flow. Friction appears wherever the fluid flow is surrounded by solid boundary. Friction can be defined as the amount of mechanical energy irreversibly converted into heat in a flow in stream. As a result of this, the total energy is always decrease in the flow direction i.e. (\(E_2 < E_1\)). Therefore \(E_1 = E + F\), where \(F\) is the energy losses due to friction.

Thus the Bernoulli’s Equation becomes:

\[ \frac{P_1}{\rho} + \frac{u_1^2}{2} + z_1 g = \frac{P_2}{\rho} + \frac{u_2^2}{2} + z_2 g + F \quad (J/\text{kg} \equiv m^2/s^2) \]  \[3.15\]

#### (iii) Pump work in Bernoulli’s equation

A pump is used in a flow system to increase the mechanical energy of the fluid. The increase being used to maintain flow of the fluid. Assume a pump is installed between the stations 1 and 2 as shown in Figure. The work supplied to the pump is shaft work (−W), the negative sign is due to work added to fluid.

Frictions occurring within the pump are: (i) Friction by fluid (ii) Mechanical friction
Since the shaft work must be discounted by these frictional force (losses) to give net mechanical energy as actually delivered to the fluid by pump (W_p).

Thus, \( W_p = \eta W_s \) where \( \eta \), is the efficiency of the pump.

Thus the modified Bernoulli’s equation for the presence of pump between the two selected points 1 and 2 becomes

\[
\frac{P_1}{\rho} + \frac{u_1^2}{2\alpha_1 g} + z_1 g + \eta W_s = \frac{P_2}{\rho} + \frac{u_2^2}{2\alpha_2 g} + z_2 g + F \quad (J / kg \equiv m^2 / s^2)
\]

3.16

By dividing each term of this equation by (g), each term will have a length units, and the equation will be:

\[
\frac{P_1}{\rho g} + \frac{u_1^2}{2\alpha_1 g} + z_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2\alpha_2 g} + z_2 + h_F \quad (m)
\]

3.17

where \( h_F = F/g \) (head loss due to friction)

**3.5 Friction in Pipes**

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of energy of fluid is lost. This loss of energy may be:

(i) Major energy losses (Skin friction): due to surface skin of the pipe

(ii) Minor energy losses (Form friction): (a) Sudden expansion or contraction pipe (b) Bends, valves and fittings (c) An obstruction in pipe

**3.5.1 Relation between Skin Friction and Wall Shear Stress**

For the flow of a fluid in short pipe of length (dL), of diameter (d), the total frictional force at the wall is the product of shear stress (\( \tau_{xw} \)) and the surface area of the pipe (\( \pi d \, dL \)). This frictional force causes a drop in pressure (\( -dP_{fs} \)). Consider a horizontal pipe as shown in Figure.

![Pipe Diagram](image)

Force balance on element (dL)

\[
\tau (\pi dL) = [P - (P + dP_{fs})] \left( \frac{\pi d^2}{4} \right)
\]

\[ -dP_{fs} = 4(\tau \rho u_x)^2 (dL/d) \rho u_x^2 \]

3.18

where \( \tau / \rho u_x^2 = \Phi = J_f = f/2 = f'/8 \)

\( \Phi \) (or \( J_f \)): Basic friction Factor

\( f \): Fanning (or Darcy) friction Factor

\( f' \): Moody friction Factor.
For incompressible fluid flowing in a pipe of constant cross-sectional area, \((u)\) is not a function of pressure or length and equation (3.18) can be integrated over a length \((L)\) to give the equation of pressure drop due to skin friction:

\[-\Delta P_f = 4f(L/d)(\rho u^2/2)\ (Pa)\]  \(3.19\)

The energy loss per unit mass \(F_s\) is then given by

\[F_s = -\Delta P_f / \rho = 4f(L/d)(u^2/2)\ (J/kg\ \text{orm}^2/\text{s}^2)\]  \(3.20\)

The head loss due to skin friction \((h_f)\) is given by

\[h_f = F_s / g = (-\Delta P_f / \rho g) = 4f(L/d)(u^2 / 2g)\ (m)\]  \(3.21\)

Equations (3.19) - (3.21) can be used for laminar and turbulent flow

\[\Delta P_f - P_2 - P_1 = -\Delta P_f - P_1 - P_2 (+\text{vc value})\]

3.5.2 Evaluation of Friction Factor in Straight Pipes

1. Velocity distribution in laminar flow

Consider a horizontal circular pipe of a uniform diameter in which a Newtonian, incompressible fluid flowing as shown in Figure:

\[
\begin{align*}
\tau_{\text{ex}} &= (2\pi r L) = (P_1 - P_2) (\pi r^2) \\
\text{for laminar flow} &\Rightarrow \tau_{\text{ex}} = -\mu \left(\frac{du}{dr}\right) \\
&\Rightarrow r (P_1 - P_2) = -\mu \left(\frac{du}{dr}\right) 2L \\
&\Rightarrow [(P_1 - P_2)/(2L \mu)] r^2/2 = u_x + C \\
\text{Boundary Condition (1)} &\quad \text{for evaluation of C} \\
\text{at } r = R &\quad u_x = 0 \\
&\Rightarrow C = [(\Delta P_f / (2L \mu))] \\
&\Rightarrow [(\Delta P_f / (4L \mu))] = u_x + [(\Delta P_f / (4L \mu))] \\
\end{align*}
\]

- \(u_x / u_{\text{max}} = [1-(r/R)^2]\) - velocity distribution (profile) in laminar flow

- \(\text{Boundary Condition (1)}\) for evaluation of \(C\)

- \(C = [(\Delta P_f / (4L \mu))]\)

- \(\frac{1}{2} = [(-\Delta P_f / (4L \mu))(1-(r/R)^2)]\) velocity distribution (profile) in laminar flow

- \(\text{Boundary Condition (2)}\) for evaluation of \(u_{\text{max}}\)

- \(\text{at } r = 0 \Rightarrow u_x = u_{\text{max}} \Rightarrow u_{\text{max}} = [(-\Delta P_f / (4L \mu))]\)

- \(u_{\text{max}} = [(-\Delta P_f / (16L \mu))]\) velocity in laminar flow
\[
\therefore \quad \frac{u_x}{u_{\text{max}}} = [1-(r/R)^2] \quad \text{--- velocity distribution (profile) in laminar flow}
\]

2. **Average (mean) linear velocity in laminar flow**

\[
Q = uA \quad \text{------------------------ (1)}
\]

Where, \((u)\) is the average velocity and \((A)\) is the cross-sectional area \(= (\pi R^2)\)

\[
dQ = u \, dA \quad \text{where} \quad u_x = u_{\text{max}}[1-(r/R)^2], \text{and} \quad dA = 2\pi \, r \, dr
\]

\[
\Rightarrow \quad dQ = u_{\text{max}}[1-(r/R)^2] \, 2\pi \, r \, dr
\]

\[
\int_0^R dQ = 2\pi u_{\text{max}} \int_0^R (r - \frac{r^3}{R^2}) \, dr = 2\pi u_{\text{max}} \left[ \frac{r^2}{2} - \frac{r^4}{4R^2} \right]_0^R
\]

\[
\Rightarrow \quad Q = \frac{u_{\text{max}}}{2} (\pi R^2) \quad \text{------------------------ (2)}
\]

By equalization of equations (1) and (2)

\[
\Rightarrow \quad u = \frac{u_{\text{max}}}{2} = \left[ (-\Delta P_{\text{f}} \, R^2)/(8L \, \mu) \right] = \left[ (-\Delta P_{\text{f}} \, d^2)/(32L \, \mu) \right]
\]

\[
\therefore \quad \Delta P_{\text{f}} = (32L \, \mu \, u) / d \quad \text{Hagen–Poiseuille equation}
\]

3. **Friction factor in laminar flow**

We have \(-\Delta P_{\text{f}} = 4f (L/d) \, (\rho u^2)/2\)\-------------------------(3)

and also \(-\Delta P_{\text{f}} = (32L \, \mu \, u) / d^2\ \quad \text{-------------------------(4)}\)

By equalization of these equations [i.e. eqs. (3) and (4)]

\[
\Rightarrow \quad (32L \, \mu \, u) / d^2 = 4f (L/d) \, (\rho u^2)/2 \Rightarrow f = 16 \, \mu / (\rho u d)
\]

\[
\therefore f = 16 / Re \quad \text{Fanning or Darcy friction factor in laminar flow.}
\]

**NOTE:** Using Moody friction factor \((f')\)

\[
\Rightarrow \quad (32L \, \mu u) / d^2 = 4(f'/8)(L/d)(\rho u^2) \Rightarrow f' = 64\mu / (\rho ud)
\]

\[
\therefore f' = 64 / Re \quad \text{- Moody friction factor}
\]
4. **Velocity distribution in turbulent flow**

The velocity, at any point in the cross-section of cylindrical pipe, in turbulent flow is proportional to the one-seventh power of the distance from the wall. This may be expressed as follows:

\[
\frac{d\nu}{\nu_{\text{max}}} = \left[1-\left(\frac{r}{R}\right)\right]^{\frac{1}{7}}
\]

Prandtl's one-seventh law equation.

**Velocity distribution (profile) in laminar flow**

5. **Average (mean) linear velocity in Turbulent flow**

\[
Q = u_{\text{max}} A
\]

\[
dQ = u_x \, dA
\]

where \( u_x = u_{\text{max}} \left[1-\left(\frac{r}{R}\right)\right]^{\frac{1}{7}}, \) and \( dA = 2\pi \, r \, dr \)

\[
\Rightarrow dQ = u_{\text{max}} \left[1-\left(\frac{r}{R}\right)\right]^{\frac{1}{7}} 2\pi \, r \, dr
\]

\[
\int_0^R dQ = 2\pi \, u_{\text{max}} \int_0^R \left[1-\left(\frac{r}{R}\right)\right]^{\frac{1}{7}} \, r \, dr
\]

Let \( M = (1-r/R) \quad \text{dM} = (-1/R) \, \text{dr} \)

or \( r = R(1-M) \quad \text{dr} = -R \, \text{dM} \)

at \( r = 0 \quad M = 1 \)

at \( r = R \quad M = 0 \)

Rearranging the integration

\[
Q = u_{\text{max}} \, 2\pi \, R^2 \int_0^1 (1-M) M^{\frac{1}{7}} (-\text{dM}) = u_{\text{max}} \, 2\pi \, R^2 \int_0^1 (M^{\frac{1}{7}} - M^{\frac{15}{7}}) \, \text{dM}
\]

\[
Q = u_{\text{max}} \, 2\pi \, R^2 \left[\frac{M^{\frac{8}{7}}}{8/7} - \frac{M^{\frac{15}{7}}}{15/7}\right]_0^1 = u_{\text{max}} \, 2\pi \, R^2 \left[\frac{7}{8} - \frac{7}{15}\right]
\]

\[
\Rightarrow Q = \frac{49}{60} u_{\text{max}} (\pi R^2)
\]

By equalization of equations (1) and (5)

\[
\therefore \overline{u} = \frac{49}{60} u_{\text{max}} \approx 0.82 u_{\text{max}}
\]

--- average velocity in turbulent flow

6. **Friction factor in Turbulent flow**

A number of expressions have been proposed for calculating friction factor in terms of or function of \( \text{Re} \). Some of these expressions are given here:

\[
f = \frac{0.079}{\text{Re}^{0.25}} \quad \text{for} \ 2,500 < \text{Re} < 100,000
\]

and,

\[
f = 4 \log(\text{Re}) - 0.4 \quad \text{for} \ 2,500 < \text{Re} < 10,000,000
\]

These equations are for **smooth pipes** in turbulent flow. For rough pipes, the ratio of \((e/d)\) acts an important role in evaluating the friction factor in turbulent flow as shown in the following equation

\[
(f/2)^{0.5} = -2.5 \ln \left[ 0.27 \frac{e}{d} + 0.885 \text{Re}^{-1} (f/2)^{0.5} \right]
\]
Table of the roughness values $e$. 

<table>
<thead>
<tr>
<th>Surface type</th>
<th>ft</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plained wood or finished concrete</td>
<td>0.00015</td>
<td>0.046</td>
</tr>
<tr>
<td>Unplained wood</td>
<td>0.00024</td>
<td>0.073</td>
</tr>
<tr>
<td>Unfinished concrete</td>
<td>0.00037</td>
<td>0.11</td>
</tr>
<tr>
<td>Cast iron</td>
<td>0.00056</td>
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<tr>
<td>Brick</td>
<td>0.00082</td>
<td>0.25</td>
</tr>
<tr>
<td>Riveted steel</td>
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</tr>
<tr>
<td>Corrugated metal</td>
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<td>1.68</td>
</tr>
<tr>
<td>Rubble</td>
<td>0.012</td>
<td>3.66</td>
</tr>
</tbody>
</table>

7. Graphical evaluation of friction factor

As with the results of Reynolds number the curves are in three regions (Figure 3.7 vol.1). At low values of Re ($Re < 2,000$), the friction factor is independent of the surface roughness, but at high values of Re ($Re > 2,500$) the friction factor vary with the surface roughness. At very high Re, the friction factor become independent of Re and a function of the surface roughness only. Over the transition region of Re from 2,000 to 2,500 the friction factor increased rapidly showing the great increase in friction factor as soon as turbulent motion established.

![Figure 3.7. Pipe friction chart $\phi$ versus $Re$ (also see fold-out in the Appendix)](image)

Example 3.3
Water flowing through a pipe of 20 cm I.D. at section 1 and 10 cm at section 2. The discharge through the pipe is 35 lit/s. The section 1 is 6 m above the datum line and section 2 is 2 m above it. If the pressure at section is 245 kPa, find the intensity of pressure at section 2. Given that \( \rho = 1000 \text{ kg/m}^3 \), \( \mu = 1.0 \text{ mPa.s} \).

**Solution**

\[ Q = 35 \text{ lit/s} = 0.035 \text{ m}^3/\text{s} \]

\[ u = \frac{Q}{A} \Rightarrow u_1 = \frac{(0.035 \text{ m}^3/\text{s})}{(0.2^2 \pi/4) \text{ m}^2} = 1.114 \text{ m/s} \]

\[ u_2 = \frac{(0.035 \text{ m}^3/\text{s})}{(0.1^2 \pi/4) \text{ m}^2} = 4.456 \text{ m/s} \]

\[ Re = \frac{\rho u d}{\mu} \Rightarrow Re_1 = \frac{(1000 \text{ kg/m}^3 \times 1.114 \text{ m/s} \times 0.2 \text{ m})}{(0.001 \text{ Pa.s})} = 222,800 \]

\[ Re_2 = \frac{(1000 \text{ kg/m}^3 \times 4.456 \text{ m/s} \times 0.1 \text{ m})}{(0.001 \text{ Pa.s})} = 445,600 \]

The flow is turbulent along the tube (i.e. \( \alpha_1 = \alpha_2 = 1.0 \)).

\[ P_1 + \frac{u_1^2}{2\alpha_1} + g z_1 + \gamma w_1 = P_2 + \frac{u_2^2}{2\alpha_2} + g z_2 + \gamma w_2 \]

\[ P_2 = \rho \left[ \frac{P_1}{\rho} + g(z_1 - z_2) + \left( \frac{u_1^2}{2\alpha_1} - \frac{u_2^2}{2\alpha_2} \right) \right] = 274.9 \text{ kPa} \]

**Example 3.4**: If the pipe is smooth and its length is 20 m, find \( P_2 \)

**Solution**

\[ Re_2 = 445,600 \]

Check the curve for smooth pipe on Figure 3.7 and determine \( f \) which is equal to 0.0034

Recall that: \( \Delta P_s = P_1 - P_2 = 4f \left( \frac{L}{d} \right) \rho u^2 \]

\[ P_2 = P_1 + 4f \left( \frac{L}{d} \right) \rho u^2 \]

\[ P_2 = 245000 + 4(0.0034) \left( \frac{20}{2} \right) \frac{1000 \times 4.456^2}{2} = 246350.204 \text{ Pa} = 246.35 \text{kPa} \]

**Example 3.5**

A conical tube of 4 m length is fixed at an inclined angle of 30° with the horizontal line and its small diameter upwards. The velocity at smaller end is \( u_1 = 5 \text{ m/s} \), while \( u_2 = 2 \text{ m/s} \) at other end. The head
losses in the tub is \[0.35 (u_1-u_2)^2/2g\]. Determine the pressure head at lower end if the flow takes place in downward direction and the pressure head at smaller end is 2 m of liquid.

**Solution:**

No information of the fluid properties. Then assume the flow is turbulent, (i.e. \(\alpha_1 = \alpha_2 = 1.0\))

\[
\frac{P_1 - P_2}{\rho g} + \frac{u_1^2}{2g} + z_1 + \frac{\eta W_s}{g} = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + z_2 + h_f
\]

\[
\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + \frac{u_1^2 - u_2^2}{2g} - 0.35 \frac{(u_1 - u_2)}{2g} - 2 m
\]

\[
= 2.0 + 2.0 + (25 - 4) / (2 \times 9.81) - 0.35(5 - 2)^2 / (2 \times 9.81) = 4.9 m
\]

**Example 3.6**

Water with density \(\rho = 998 \text{ kg/m}^3\), is flowing at steady mass flow rate through a uniform-diameter pipe. The entrance pressure of the fluid is 68.9 kPa in the pipe, which connects to a pump, which actually supplies 155.4 J/kg of fluid flowing in the pipe. The exit pipe from the pump is the same diameter as the inlet pipe. The exit section of the pipe is 3.05 m higher than the entrance, and the exit pressure is 137.8 kPa. The Reynolds number in the pipe is above 4,000 in this system. Calculate the frictional loss (F) in the pipe system.

**Solution:**

Setting the datum line at \(z_1\) thus, \(z_1 = 0, z_2 = 3.05 m\)

\[
\frac{P_1}{\rho} + \frac{u_1^2}{2g} + z_1 + \frac{\eta W_s}{g} = \frac{P_2}{\rho} + \frac{u_2^2}{2g} + z_2 + F
\]

\[
\Rightarrow F = \frac{P_1 - P_2}{\rho} + \frac{\eta W_s}{g} - g z_2
\]

\[
= (68.9 - 137.8) \times 1000 / 998 + 155.4 - 9.81(3.05)
\]

\[
= 50.5 \text{ J/kg or m}^2/\text{s}^2
\]

**3.5.3 Form Friction**

Skin friction loss in straight pipe flow is calculated by using the Fanning friction factor \((f)\). However, if the velocity of the fluid is changed in direction or magnitude, additional friction losses occur. This results from additional turbulence, which develops because of vertices and other factors.
1- Sudden Expansion (Enlargement) Losses

If the cross section of a pipe enlarges gradually, very little or no extra losses are incurred. If the change is sudden, as that in Figure, it results in additional losses due to eddies formed by the jet expanding in the enlarged section. This friction loss can be calculated by the following for laminar or turbulent flow in both sections, as:

\[
\frac{u_1 A_1}{2} - \frac{u_2 A_2}{2} = \frac{u_1}{2} \left[ 1 - \left( \frac{A_1}{A_2} \right) \right]
\]

\[
F_f = \frac{u_1^2}{2} - \frac{u_2^2}{2} - u_1 u_2
\]

\[
\therefore F_f = K_e \frac{u_2^2}{2} \quad \text{where} \quad K_e = \left[ 1 - \left( \frac{A_1}{A_2} \right) \right]
\]

The change in pressure \(-\Delta P_f\) is therefore given by:

\[
-\Delta P_f = \rho \frac{(u_1 - u_2)^2}{2} = \frac{\rho u_1^2}{2} \left[ 1 - \left( \frac{A_1}{A_2} \right) \right]^2
\]

The loss of head \(h_f\) is given by:

\[
h_f = \frac{(u_1 - u_2)^2}{2g}
\]

Example 3.7

Water flows at 7.2 \(m^3/h\) through a sudden enlargement from a 40 mm to a 50 mm diameter pipe. What is the loss in head?

**Solution**

Velocity in 50 mm pipe = \(\frac{7.2}{(\pi/4)(50 \times 10^{-3})^2} = 1.02 \text{ m/s}\)

Velocity in 40 mm pipe = \(\frac{7.2}{(\pi/4)(40 \times 10^{-3})^2} = 1.59 \text{ m/s}\)

The head lost is given by

\[
h_f = \frac{(u_1 - u_2)^2}{2g} = \frac{(1.59 - 1.02)^2}{2 \times 9.81} = 0.0165 \text{ m water}
\]

or: \(16.5 \text{ mm water}\)

2. Sudden Contraction Losses
As shown in Figure, the effective area for flow gradually decreases as a sudden contraction is approached and then continues to decrease, for a short distance, to what is known as the *vena contracta*. After the *vena contracta* the flow area gradually approaches that of the smaller pipe. As the fluid moves towards the *vena contracta* it is accelerated and pressure energy is converted into kinetic energy; this process does not give rise to eddy formation and losses are very small. Beyond the *vena contracta*, however, the velocity falls as the flow area increases and conditions are equivalent to those for a sudden enlargement. The expression for the loss at a sudden enlargement can therefore be applied for the fluid flowing from the *vena contracta* to some section a small distance downstream, where the whole of the cross-section of the pipe is available for flow.

The frictional loss per unit mass of fluid is then given by:

\[
F = \frac{(u_c - u_2)^2}{2} = \frac{u_2^2}{2} \left(\frac{u_c}{u_2} - 1\right)^2
\]

Denoting the ratio of the area at section \(C\) to that at section \(2\) by a coefficient of contraction \(C_c\):

\[
F = \frac{u_2^2}{2} \left(\frac{1}{C_c} - 1\right)^2
\]

\[
F_c = K_c \frac{u_2^2}{2} \quad K_c = \left[\frac{1}{C_c} - 1\right]^2
\]

Thus the change in pressure \(\Delta P_f\) is \(-\rho u^2/2\)[(1/\(C_c\)) - 1] and the head lost is:

\[
\frac{u_2^2}{2g} \left[\frac{1}{C_c} - 1\right]^2
\]

\(C_c\) varies from about 0.6 to 1.0 as the ratio of the pipe diameters varies from 0 to 1. For a common value of \(C_c\) of 0.67:

\[
F = \frac{u_2^2}{8}
\]

It may be noted that the maximum possible frictional loss which can occur at a change in cross-section is equal to the entire kinetic energy of the fluid.

**Example 3.8**
A pipe of diameter 225mm is attached to a 150 mm pipe by means of a flange in such a manner that the axes of the two pipes are in a straight line. Water flows through the arrangement at the rate of 0.05 m³/s. The pressure loss at the transition as indicated by differential gauge length on a water-mercury manometer connected between the two pipes equals 35mm. Calculate: (i) the loss of head due to contraction, and (ii) the coefficient of contraction.

**Solution**

**Diameter of large pipe**, \( d_1 = 225\text{mm} = 0.225\text{m} \)

**Area**, \( A_1 = \frac{\pi}{4} \times 0.225^2 = 0.03976\text{m}^2 \)

**Diameter of small pipe**, \( d_2 = 150\text{mm} = 0.150\text{m} \)

**Area**, \( A_2 = \frac{\pi}{4} \times 0.150^2 = 0.01767\text{m}^2 \)

**Discharge**, \( Q = 0.05\text{ m}^3/\text{s} \)

**Reading of the differential gauge**, \( h = 35\text{mm} = 0.035\text{m} \)

(i) **Loss of head due to contraction** \( h_c \):

When the water-mercury manometer is connected across the contracted transition, then

\[
\frac{P_1 - P_2}{\rho g} = h \left( \frac{S_m}{S_w} - 1 \right)
\]

where \( S_m = \text{specific gravity of mercury} = 13.6 \), \( S_w = \text{specific gravity of water} = 1 \)

\[
\frac{P_1 - P_2}{\rho g} = h \left( \frac{13.6}{1} - 1 \right) = 0.035 \times 13.6 = 0.44\text{m}
\]

Let \( u_1 \) and \( u_2 \) be velocities of flow in the large and small diameter pipes, respectively. Then:

\[
\frac{Q}{A_1} = \frac{0.05}{0.03976} = 1.26\text{m/s}
\]

\[
\frac{Q}{A_2} = \frac{0.05}{0.01767} = 2.83\text{m/s}
\]

Then applying the Bernoulli's Equation with frictional loss:

\[
\frac{(P_1 - P_2)}{\rho g} + \frac{(u_1^2 - u_2^2)}{2g} + (z_1 - z_2) = h_c
\]

Note the \( h_c = \text{head loss due to contraction} \) and \( z_1 = z_2 \) (horizontal pipe)

\[
h_c = 0.441 + \frac{(1.26^2 - 2.83^2)}{2 \times 9.81} = 0.114\text{m of water}
\]

(ii) **The coefficient of contraction** \( C_c \)
\[ h_c = \frac{u^2}{2g} \left( \frac{1}{C_c} - 1 \right)^2 \]

\[ 0.114 = \frac{2.83^2}{2 \times 9.81} \left( \frac{1}{C_c} - 1 \right)^2 \Rightarrow C_c = 0.65 \]

3- Losses in Fittings and Valves

Pipe fittings and valves also disturb the normal flow lines in a pipe and cause additional friction losses. In a short pipe with many fittings, the friction losses from these fittings could be greater than in the straight pipe. Some representative figures are given in Table 3.3 for the friction losses in various pipe fittings for turbulent flow of fluid, and are expressed in terms of the equivalent length of straight pipe with the same resistance, and as the number of velocity heads \( (u^2/2g) \) lost. Considerable variation occurs according to the exact construction of the fittings. Typical pipe-fittings are shown in Figures below.

<table>
<thead>
<tr>
<th>Table 3.3. Friction losses in pipe fittings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>45° elbows (a)*</td>
</tr>
<tr>
<td>90° elbows (standard radius) (b)</td>
</tr>
<tr>
<td>90° square elbows (c)</td>
</tr>
<tr>
<td>Entry from leg of T-piece (d)</td>
</tr>
<tr>
<td>Entry into leg of T-piece (d)</td>
</tr>
<tr>
<td>Unions and couplings (e)</td>
</tr>
<tr>
<td>Globe valves fully open</td>
</tr>
<tr>
<td>Gate valves: fully open</td>
</tr>
<tr>
<td>( \frac{1}{2} ) open</td>
</tr>
<tr>
<td>( \frac{1}{3} ) open</td>
</tr>
<tr>
<td>( \frac{1}{4} ) open</td>
</tr>
</tbody>
</table>

*See Figure
Evaluation of the friction loss in valves and fittings involves the determination of the appropriate loss coefficient \( K_f \), which in turn defines the energy loss per unit mass of fluid:

\[ F_f = K_f \frac{u_1^2}{2} \]

The basis for the equivalent L/D method is the assumption that there is some length of pipe \( (L_{eq}) \) that has the same friction loss as that which occurs in the fitting, at a given (pipe) Reynolds number. Thus, the fittings are conceptually replaced by the equivalent additional length of pipe that has the same friction loss as the fitting:

\[ F_f = 4f \frac{u_1^2}{2} \sum \left( \frac{L}{d} \right)_{eq} \]

### 3.5.4 Total Friction Losses

The frictional losses from the friction in the straight pipe (skin friction), enlargement losses, contraction losses, and losses in fittings and valves are all incorporated in F term in mechanical energy balance equation (modified Bernoulli’s equation), so that,

\[
F = 4f \frac{L u^2}{d^2} + K_e \frac{u^2}{2} + K_c \frac{u_1^2}{2} + K_f \frac{u^2}{2}
\]

If all the velocity \( u, u_1 \) and \( u_2 \) are the same, then this equation becomes, for this special case;

\[
F = \left[ 4f \frac{L}{d} + K_e + K_c + K_f \right] \frac{u^2}{2}
\]

If equivalent length of the straight pipe for the losses in fittings and/or valves, then this equation becomes:

\[
F = \left[ 4f \left( \frac{L}{d} + \sum \frac{Le}{d} \right) + K_e + K_c \right] \frac{u^2}{2}
\]
Example 3.9

2.27 m³/h water at 320 K is pumped in a 40 mm i.d. pipe through a distance of 150 m in a horizontal direction and then up through a vertical height of 10 m. In the pipe there is a control valve for which the friction loss may be taken as equivalent to 200 pipe diameters and also other pipe fittings equivalent to 60 pipe diameters. Also in the line is a heat exchanger across which there is a loss in head of 1.5 m of water. If the main pipe has a roughness of 0.2 mm, what power must be supplied to the pump if it is 60 per cent efficient?

Solution

Relative roughness: \( \frac{e}{d} = \left( \frac{0.2}{40} \right) = 0.005 \)

Viscosity at 320 K: \( \mu = 0.65 \text{ mN s/m}^2 \) or \( 0.65 \times 10^{-3} \text{ N s/m}^2 \)

Flowrate = 2.27 m³/h = \( 6.3 \times 10^{-4} \text{ m}^3/\text{s} \)

Area for flow = \( \frac{\pi (40 \times 10^{-3})^2}{4} = 1.26 \times 10^{-3} \text{ m}^2 \)

Thus:

Velocity = \( \frac{6.3 \times 10^{-4}}{1.26 \times 10^{-3}} = 0.50 \text{ m/s} \)

and:

\( \text{Re} = \frac{(40 \times 10^{-3} \times 0.50 \times 1000)}{(0.65 \times 10^{-3})} = 30,770 \)

giving: \( \frac{\tau}{\rho u^2} = 0.004 \) (from Figure 3.7)

Equivalent length of pipe = \( 150 + 10 + (260 \times 40 \times 10^{-3}) = 170.4 \text{ m} \)

\( k_f = 4 \frac{R}{\rho u^2} \frac{1}{d} \frac{u^2}{g} \)

\( = 4 \times 0.004 \left( \frac{170.4}{40 \times 10^{-3}} \right) \left( \frac{0.5^2}{9.81} \right) \)

\( = 1.74 \text{ m} \)

Total head to be developed = \( (1.74 + 1.5 + 10) = 13.24 \text{ m} \)

Mass throughput = \( (6.3 \times 10^{-4} \times 1000) = 0.63 \text{ kg/s} \)

Power required = \( (0.63 \times 13.24 \times 9.81) = 81.8 \text{ W} \)

Since the pump efficiency is 60 per cent, the power required = \( \frac{81.8}{0.60} = 136.4 \text{ W} \) or \( 0.136 \text{ kW} \)

The kinetic energy head, \( u^2/2g \) amounts to \( 0.5^2/(2 \times 9.81) = 0.013 \text{ m} \), and this may be neglected.
EXERCISE

3.1 A petroleum fraction is pumped 2 km from a distillation plant to storage tank through a mild steel pipeline, 150 mm I.D. at 0.04 m$^3$/s rate. What is the pressure drop along the pipe and the power supplied to the pumping unit if it has an efficiency of 50%. The pump impeller is eroded and the pressure at its delivery falls to one half. By how much is the flow rate reduced? Take that: sp.gr. = 0.705, $\mu$= 0.5m Pa.s $e = 0.004$ mm.

3.2 Water at 20°C being pumped from a tank to an elevated tank at the rate of 0.005 m$^3$/s. All the piping in the Figure below is 4” Schedule 40 pipe. The pump has an efficiency of $\eta = 0.65$. Calculate the kW power needed for the pump. $e = 4.6 \times 10^{-5}$ m $\rho = 998.2$ kg/m, $\mu = 1.005 \times 10^{-3}$ Pa.s

3.3 98% H$_2$SO$_4$ is pumped at 1.25 kg/s through a 25 mm inside diameter pipe, 30 m long, to a reservoir 12 m higher than the feed point. Calculate the pressure drop in the pipeline. Take that $\rho = 1840$ kg/m$^3$, $\mu = 25$ mPa.s, $e = 0.05$ mm.

3.4 Water is being discharged, from a reservoir, through a pipe 4 km long and 50 cm I.D. to another reservoir having water level 12.5 m below the first reservoir. It is required to feed a third reservoir, whose level is 15 m below the first reservoir, through a pipe line 1.5 km long to be connected to the pipe at distance of 1.0 km from its entrance. Find the diameter of this new pipe, so that the flow into both the reservoirs may be the same.
5.1 Introduction

Pumps are devices for supplying energy or head to a flowing fluid in order to overcome head losses due to friction and also if necessary, to raise liquid to a higher level. For the pumping of liquids or gases from one vessel to another or through long pipes, some form of mechanical pump is usually employed. The energy required by the pump will depend on the height through which the fluid is raised, the pressure required at delivery point, the length and diameter of the pipe, the rate of flow, together with the physical properties of the fluid, particularly its viscosity and density.

The pumping of liquids such as sulphuric acid or petroleum products from bulk store to process buildings, or the pumping of fluids round reaction units and through heat exchangers, are typical illustrations of the use of pumps in the process industries. On the one hand, it may be necessary to inject reactants or catalyst into a reactor at a low, but accurately controlled rate, and on the other to pump cooling water to a power station or refinery at a very high rate. The fluid may be a gas or liquid of low viscosity, or it may be a highly viscous liquid, possibly with non-Newtonian characteristics. It may be clean, or it may contain suspended particles and be very corrosive. All these factors influence the choice of pump. Because of the wide variety of requirements, many different types are in use including centrifugal, piston, gear, screw, and peristaltic pumps, though in the chemical and petroleum industries the centrifugal type is by far the most important.

The factors that influence the choice of pump for a particular operation include:

(i) **The quantity of liquid to be handled**: This primarily affects the size of the pump and determines whether it is desirable to use a number of pumps in parallel.

(ii) **The head against which the liquid is to be pumped**: This will be determined by the difference in pressure, the vertical height of the downstream and upstream reservoirs and by the frictional losses which occur in the delivery line. The suitability of a centrifugal pump and the number of stages required will largely be determined by this factor.

(iii) **The nature of the liquid to be pumped**: For a given throughput, the viscosity largely determines the friction losses and hence the power required. The corrosive nature will determine the material of construction both for the pump and the packing. With suspensions, the clearances in the pump must be large compared with the size of the particles.

(iv) **The nature of the power supply**: If the pump is to be driven by an electric motor or internal combustion engine, a high-speed centrifugal or rotary pump will be preferred as it can be coupled directly to the motor. Simple reciprocating pumps can be connected to steam or gas engines.

(5) If the pump is used only intermittently, corrosion problems are more likely than with continuous working.
The head imparted to a flowing liquid by a pump is known as the total head ($\Delta h$). If a pump is placed between points 1 and 2 in a pipeline, the head for steady flow are related by:

$$\Delta h = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + z_2 - \frac{P_1}{\rho g} - \frac{u_1^2}{2g} - z_1 - h_f$$

$$\Rightarrow \Delta h = \frac{\Delta P}{\rho g} + \frac{\Delta u^2}{2\alpha g} + \Delta z + h_f$$

4.2 System Heads

The important heads to consider in a pumping system are:

1- Suction head
2- Discharge head
3- Total head
4- Net Positive Suction Head (NPSH)

The following definitions are given in reference to typical pumping system shown in Figure 4.1, where the datum line is the center line of the pump

1- Suction head ($h_s$)

$$h_s = z_s + \frac{P_s}{\rho g} - (h_f)_s$$

2- Discharge head ($h_d$)

$$h_d = z_d + \frac{P_d}{\rho g} + (h_f)_d$$

3- Total head ($\Delta h$)

The total head ($\Delta h$), which is required to impart to the flowing liquid is the difference between the discharge and suction heads. Thus,

$$\Delta h = h_d - h_s$$

$$\Rightarrow \Delta h = (z_d - z_s) + \left(\frac{P_d - P_s}{\rho g}\right) + [(h_f)_d + (h_f)_s]$$

where,

$$(h_f)_d = 4f_d\left(\frac{L}{d} + \sum \frac{Le}{d}\right)\frac{u_d^2}{2g}$$

$$(h_f)_s = 4f_s\left(\frac{L}{d} + \sum \frac{Le}{d}\right)\frac{u_s^2}{2g}$$

The suction head ($h_s$) decreases and the discharge head ($h_d$) increases with increasing liquid flow rate because of the increasing value of the friction head loss terms ($h_f)_s$ and ($h_f)_d$. Thus the total head ($\Delta h$)
which the pump is require to impart to the flowing liquid increases with increasing the liquid pumping rate. Note that if the liquid level on the suction side is below the center line of the pump, \( z \) is negative.

4. Net positive suction head (NPSH)

Available net positive suction head

\[
NPSH = z_s + \left( \frac{P_s - P_v}{\rho g} \right) - (h_F)_s
\]

This equation (4.6) gives the head available to get the liquid through the suction piping.

\( P_v \), the vapor pressure of the liquid being pumped at the particular temperature in question.

The available net positive suction head (NPSH) can also be written as:

\[
NPSH = h_i - \frac{P_v}{\rho g}
\]

The available NPSH in a system should always be positive i.e. the suction head should always be capable of overcoming the vapor pressure (\( P_v \)) since the frictional head loss (\( h_F \)) increases with increasing pumping rate.

At the boiling temperature of the liquid \( P_s \) and \( P_v \) are equal and the available NPSH becomes \([z_s(h_F)_s]\). In this case no suction lift is possible since \( z_s \) must be positive. If the term \((P_s - P_v)\) is sufficiently large, liquid can be lifted from below the centerline of the pump. In this case \( z_s \) is negative.

From energy consideration it is immaterial whether the suction pressure is below atmospheric pressure or well above it, as long as the fluid remains liquid. However, if the suction pressure is less than the vapour pressure at the pumping temperature, vaporisation will occur and the pump may not be capable of developing the required suction head. Moreover, if the liquid contains gases, these may come out of solution giving rise to pockets of gas. This phenomenon is known as cavitation and may result in mechanical damage to the pump as the bubbles collapse. The tendency for cavitation to occur is accentuated by any sudden changes in the magnitude or direction of the velocity of the liquid in the pump. The onset of cavitation is accompanied by a marked increase in noise and vibration as the vapour bubbles collapse, and also a loss of head.

To avoid cavitations, the pressure at the pump inlet must exceed the vapor pressure by a certain value, called the “Net Positive Suction Head (NPSH)”. The required values of NPSH is about 2-3 m H\(_2\)O for small pump; but it increases with pump capacity and values up to 15 m H\(_2\)O are recommended for very large pump.

4.3 Power Requirement

The power requirement to the pump drive from an external source is denoted by \( (P) \). It is calculated from \( W_s \) by:

\[
P = \dot{m}W_s = \frac{Q \Delta P}{\eta} = \frac{Q \Delta h}{\eta} \frac{\rho g}{\eta} = \dot{m} \Delta hg
\]
The mechanical efficiency ($\eta$) decreases as the liquid viscosity and the frictional losses increase. The mechanical efficiency is also decreased by power losses in gear, bearing, seals, etc. These losses are not proportional to pump size. Relatively large pumps tend to have the best efficiency whilst small pumps usually have low efficiencies. Furthermore, high-speed pumps tend to be more efficient than low-speed pumps. In general, high efficiency pumps have high NPSH requirements.

4.4 Types of Pumps

Pumps can be classified into two categories:
(i) Positive displacement pumps.
(ii) Dynamic/Kinetic pumps

4.4.1 Positive Displacement Pumps

The positive displacement pump is one in which the liquid discharge is directly proportional to the displacement of the moving elements and independent of the fluid inlet pressure. They are employed for constant delivery rates and where delivery against high pressures is required. They are subdivided into three categories: (i) the reciprocating pump and (ii) the rotary pump (iii) pneumatic pumps.

(i) Reciprocating Pump

The moving element in the reciprocating pump is the piston which moves back and forth in a cylinder in response to the rotary motion of a crank or flywheel. During the suction stroke, the pump cylinder fills with fresh liquid, and the discharge stroke displaces it through a check valve into the discharge line. Reciprocating pumps can develop very high pressures. Plunger, piston and diaphragm pumps are under these type of pumps.

(ii) Rotary Pump

The moving element in the rotary pump is the impeller which rotates in the casing. The pump rotor of rotary pumps displaces the liquid either by rotating or by a rotating and orbiting motion. It mechanisms consisting of a casing with closely fitted cams, lobes, or vanes, that provide a means for conveying a fluid. Vane, gear, and lobe pumps are examples of positive displacement rotary pumps

(iii) Pneumatic Pump

Compressed air is used to move the liquid in pneumatic pumps. In pneumatic ejectors, compressed air displaces the liquid from a gravity-fed pressure vessel through a check valve into
the discharge line in a series of surges spaced by the time required for the tank or receiver to fill again.

4.4.2 Dynamic /Kinetic Pumps
Dynamic pumps impart velocity and pressure to the fluid as it moves past or through the pump impeller and, subsequently, convert some of that velocity into additional pressure. It is also called Kinetic pumps. Kinetic pumps are subdivided into the following groups: centrifugal pumps, peripheral and special pumps.

(i) Centrifugal Pumps
The centrifugal pump is by far the most widely used type in the chemical and petroleum industries. It will pump liquids with very wide-ranging properties and suspensions with a high solids content including, for example, cement slurries, and may be constructed from a very wide range of corrosion resistant materials. The whole pump casing may be constructed from plastics such as polypropylene or it may be fitted with a corrosion-resistant lining. Because it operates at high speed, it may be directly coupled to an electric motor and it will give a high flowrate for its size. A centrifugal pump is a rotating machine in which flow and pressure are generated dynamically. The energy changes occur by virtue of two main parts of the pump, the impeller and the volute or casing. The function of the casing is to collect the liquid discharged by the impeller and to convert some of the kinetic (velocity) energy into pressure energy.

A diagrammatic representation of various types of pump is presented in Figure 4.2
The advantages and disadvantages of the centrifugal pump

The main advantages are:

1. It is simple in construction and can be made in a wide range of materials.
2. There is a complete absence of valves.
3. It operates at high speed (up to 100 Hz) and therefore, can be coupled directly to an electric motor. In general, the higher the speed the smaller the pump and motor for a given duty.
4. It gives a steady delivery.
5. Maintenance costs are lower than for any other type of pump.
6. No damage is done to the pump if the delivery line becomes blocked, provided it is not ran in this condition for a prolonged period.
7. It is much smaller than other pumps of equal capacity. It can, therefore, be made into a sealed unit with the driving motor, and immersed in the suction tank.
8. Liquids containing high proportions of suspended solids are readily handled.

The main disadvantages are:

1. The single-stage pump will not develop a high pressure. Multistage pumps will develop greater heads but they are very much more expensive and cannot readily be made in corrosion-resistant material because of their greater complexity. It is generally better to use very high speeds in order to reduce the number of stages required.
2. It operates at a high efficiency over only a limited range of conditions: this applies especially to turbine pumps.
3. It is not usually self-priming.
4. If a non-return valve is not incorporated in the delivery or suction line, the liquid will run back into the suction tank as soon as the pump stops.
5. Very viscous liquids cannot be handled efficiently.
4.5 Priming the Pump

The theoretical head developed by a centrifugal pump depends on the impeller speed, the radius of the impeller, and the velocity of the fluid leaving the impeller. If these factors are constant, the developed head is the same for fluids of all densities and is the same for liquids and gases. A centrifugal pump trying to operate on air, then can neither draw liquid upward from an initially empty suction line nor force liquid to a full discharge line. Air can be displaced by priming the pump.

For example, if a pump develops a head of 100 ft and is full of water, the increase in pressure is $[100 \text{ ft} \left(62.3 \text{ lb/ft}^3\right) \left(\text{ft}^2 / 144 \text{ in}^2\right)] = 43 \text{ psi} \left(2.9 \text{ atm}\right)$. If full of air the pressure increase is about 0.05 psi (0.0035 atm).

4.6 Operating Characteristics
The operating characteristics of a pump are conveniently shown by plotting the head (h), power (P), efficiency (η), and sometimes required NPSH against the flow (or capacity) (Q) as shown in Figure 4.3. These are known as characteristic curves of the pump. It is important to note that the efficiency reaches a maximum and then falls, whilst the head at first falls slowly with Q but eventually falls off rapidly. The optimum conditions for operation are shown as the duty point, i.e. the point where the head curve cuts the ordinate through the point of maximum efficiency.

Characteristic curves have a variety of shapes depending on the geometry of the impeller and pump casing. Pump manufacturers normally supply the curves only for operation with water. In a particular system, a centrifugal pump can only operate at one point on the Δh against Q curve and that is the point where the Δh against Q curve of the pump intersect with the Δh against Q curve of the system as shown in Figure 4.4.

The system total head at a particular liquid flow rate

\[ \Delta h = (z_d - z_s) + \left( \frac{P_d - P_s}{\rho g} \right) + \left[ (h_f)_d + (h_f)_s \right] \]

where,

\[ (h_f)_d = 4 \int \frac{L}{d} \sum \frac{L_e}{d} \] \[ u_d^2 \]

\[ (h_f)_s = 4 \int \frac{L}{d} \sum \frac{L_e}{d} \] \[ u_s^2 \]

For the same pipe type and diameter for suction and discharge lines:

\[ \Delta h = \Delta z + \frac{\Delta P}{\rho g} + 4 \int \left( \frac{L}{d} + \sum \frac{L_e}{d} \right) \left( \frac{L}{d} + \sum \frac{L_e}{d} \right) \] \[ u \] \[ \frac{Q}{(\pi / 4 d^2)} \]

but \[ u = \frac{Q}{(\pi / 4 d^2)} \]

\[ \Rightarrow \Delta h = \Delta z + \frac{\Delta P}{\rho g} + 4 \int \left( \frac{L}{d} + \sum \frac{L_e}{d} \right) + \left( \frac{L}{d} + \sum \frac{L_e}{d} \right) \left( \frac{Q}{(\pi / 4 d^2)} \right) \]

Example 4.1
A petroleum product is pumped at a rate of $2.525 \times 10^{-3} \text{ m}^3/\text{s}$ from a reservoir under atmospheric pressure to 1.83 m height. If the pump 1.32 m height from the reservoir, the discharge line diameter is 4 cm and the pressure drop along its length 3.45 kPa. The gauge pressure reading at the end of the discharge line 345 kPa. The pressure drop along suction line is 3.45 kPa and pump efficiency $\eta = 0.6$ calculate: (i) The total head of the system $\Delta h$. (ii) The power required for pump. (iii) The NPSH

Take that: the density of this petroleum product $\rho = 879 \text{ kg/m}^3$, the dynamic viscosity $\mu = 6.47 \times 10^{-4}$ Pa.s, and the vapor pressure $P_v = 24.15$ kPa.

**Solution:**

(i)

$$\Delta h = (z_d - z_s) + \left( \frac{P_d - P_s}{\rho g} \right) + [(h_F)_d + (h_F)_s] + \frac{\Delta u^2}{2g}$$

$$u_s = 0$$

$$u_d = \frac{(2.525 \times 10^{-3} \text{ m}^3/\text{s})/ (\pi/4 \times 0.04^2)}{Q/A} = 2 \text{ m/s}$$

$$Re_d = \frac{879 \times 2 \times 0.04}{6.47 \times 10^{-4}}/1.087 \times 10^5$$

The pressure drop in suction line 3.45 kPa

$$\Rightarrow (h_F)_s = 3.45 \times 10^3/(879 \times 9.81) = 0.4 \text{ m}$$

And in discharge line is also 3.45 kPa $\Rightarrow (h_F)_d = 0.4 \text{ m}$

The kinetic energy term $= 2^2/(2 \times 9.81) = 0.2 \text{ m}$

The pressure at discharge point = gauge + atmospheric pressure = 345 + 101.325 = 446.325 kPa

The difference in pressure head between discharge and suction points is

$$\Delta z = 1.83 \text{ m}$$

$$\Rightarrow \Delta h = 40 \text{ m} + 1.83 \text{ m} + 0.2 \text{ m} + 0.4 \text{ m} + 0.4 \text{ m} = 42.83 \text{ m}$$

(ii)

$$P = \frac{Q \Delta P}{\eta} = \frac{Q \Delta h \rho g}{\eta} = [(2.525 \times 10^{-3} \text{ m}^3/\text{s})(42.83 \text{ m})(879 \text{ kg/m}^3)(9.81 \text{ m/s}^2)]/0.6$$

$$\Rightarrow P = 1.555 \text{ kW}$$

(iii)

$$NPSH = z_s - \left( \frac{P_s - P_v}{\rho g} \right) - (h_F)_s$$

$$= (-1.32) + (1.01325 \times 10^6 - 24150)/(879 \times 9.81) - 0.4 \text{ m}$$

$$= 7.23 \text{ m}$$

**Example 4.2**
It is required to pump cooling water from storage pond to a condenser in a process plant situated 10 m above the level of the pond. 200 m of 74.2 mm i.d. pipe is available and the pump has the characteristics given below. The head loss in the condenser is equivalent to 16 velocity heads based on the flow in the 74.2 mm pipe. If the friction factor $\Phi = 0.003$, estimate the rate of flow and the power to be supplied to the pump assuming $\eta = 0.5$

<table>
<thead>
<tr>
<th>$Q$ (m$^3$/s)</th>
<th>0.0028</th>
<th>0.0039</th>
<th>0.005</th>
<th>0.0056</th>
<th>0.0059</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta h$ (m)</td>
<td>23.2</td>
<td>21.3</td>
<td>18.9</td>
<td>15.2</td>
<td>11.0</td>
</tr>
</tbody>
</table>

**Solution:**

\[
\Delta h = \Delta z + \frac{\Delta P}{\rho g} + \frac{\Delta u^2}{2g} + (h_r)_{a} + (h_r)_{s} + (h_r)_{\text{condenser}}
\]

\[(h_r)_{a} = 4 \frac{fL}{d} \frac{u^2}{2g} = 4(0.0006)(200/0.0742)(u^2/2g) = 3.3 \text{ m}^2\]

\[(h_r)_{\text{condenser}} = 16 \frac{u^2}{2g} = 0.815 \text{ m}^2\]

\[u = \frac{Q}{A} = 231.17Q\]

\[\Rightarrow \Delta h = 10 + (0.815 + 3.3)(231.17Q)^2 = 10 + 2.2 \times 10^5 Q^2\]

To draw the system curve:

From Figure

\[Q = 0.0054 \text{ m}^3/\text{s}\]
\[\Delta h = 16.4 \text{ m}\]

Power required for pump $
\frac{Q \Delta h \rho g}{\eta} = (0.0054)(16.4)(1000)(9.81)/0.5$

\[= 17.375 \text{ kW}\]

**Example 4.3**

A pump take brine solution at a tank and transport it to another in a process plant situated 12 m above the level in the first tank. 250 m of 100 mm i.d. pipe is available sp.gr. of brine is 1.2 and $\mu = 1.2$ cp.
The absolute roughness of pipe is 0.04 mm and \( f = 0.0065 \). Calculate (i) the rate of flow for the pump (ii) the power required for pump if \( \eta = 0.65 \). (iii) if the vapor pressure of water over the brine solution at 86°F is 0.6 psia, calculate the NPSH available, if suction line length is 30 m.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
Q (m^3/s) & 0.0056 & 0.0076 & 0.01 & 0.012 & 0.013 \\
\Delta h (m) & 25 & 24 & 22 & 17 & 13 \\
\hline
\end{array}
\]

**Solution:**

(i) \( \Delta h = \Delta z + \frac{\Delta P}{\rho g} + \frac{\Delta P}{2g \alpha (h_p)_{es}} \)

\[
u = Q/A = 127.33 \, Q
\]

\[
(h_p)_{es} = 4f \frac{L}{d} \frac{u^2}{2g} = 4(0.0065)(250/0.1)(127.33 \, Q)^2/2g
\]

\[
= 53.707 \times 10^3 \, Q^2
\]

\[\Rightarrow \Delta h = 12 + 53.707 \times 10^3 \, Q^2\]

To draw the system curve

\[
\begin{array}{c|c|c|c|c|c|}
Q (m^3/h) & 0.005 & 0.007 & 0.009 & 0.011 & 0.013 \\
\Delta h (m) & 13.34 & 14.63 & 16.35 & 18.5 & 21.08 \\
\hline
\end{array}
\]

From Figure

\[
Q = 0.0114 \, m^3/s
\]

\[
\Delta h = 18.9 \, m
\]

(ii) Power required for pump

\[
\frac{Q \Delta h \rho g}{\eta} = (0.0114)(18.9) \]

\[
(1200)(9.81)/0.65 = 3.9 \, kW
\]

(iii)

\[
NPSH = z_s + \left( \frac{P_s - P_v}{\rho g} \right) - (h_p)
\]

\[
u = Q/A = 0.0114/(\pi/4 \, 0.1^2) = 1.45 \, m/s
\]

For datum line passes through the centerline of the pump \( (z_s = 0) \)

\[
(h_p) = 4f \frac{L}{d} \frac{u^2}{2g} = 4(0.0065)(30/0.1)(1.45)^2/2g = 0.84 \, m
\]

\[\Rightarrow NPSH = (101.325 \times 10^3 - 0.6 \, psi \, 101.325 \times 10^3 \, Pa/14.7 \, psi)/(1200 \times 9.81) - 0.84
\]

\[= 7.416 \, m
\]

4.7 Centrifugal Pump Relations

The power (P) required in an ideal centrifugal pump can be expected to be a function of the liquid density (\( \rho \)), the impeller diameter (D), and the rotational speed of the impeller (N). If the relationship is assumed to be given by the equation,
\[ P_F = c \rho a^b N^c \]  \hspace{1cm} \text{(1)}

then it can be shown by dimensional analysis that
\[ P_F = c_1 \rho N^3 D^3 \]  \hspace{1cm} \text{(2)}

where, \( c_1 \) is a constant which depends on the geometry of the system.

The power \( (P_F) \) is also proportional to the product of the volumetric flow rate \( (Q) \) and the total head \( (\Delta h) \) developed by the pump.
\[ P_F = c_2 Q \Delta h \]  \hspace{1cm} \text{(3)}

where, \( c_2 \) is a constant.

The volumetric flow rate \( (Q) \) and the total head \( (\Delta h) \) developed by the pump are:
\[ Q = c_3 N D^3 \]  \hspace{1cm} \text{(4)}
\[ \Delta h = c_4 N^2 D^2 \]  \hspace{1cm} \text{(5)}

where, \( c_3 \) and \( c_4 \) are constants.

Equation (5) could be written in the following form,
\[ \Delta h^{3/2} = c_4^{3/2} N^3 D^3 \]  \hspace{1cm} \text{(6)}

Combine equations (4) and (6) [ eq. (4) divided by eq. (6)] to give;
\[ \frac{Q}{\Delta h^{3/2}} = \frac{c_3}{c_4^{3/2}} N \frac{1}{3} \Rightarrow \frac{Q^{2/3}}{\Delta h^{1/2}} = \text{const.} \]  \hspace{1cm} \text{(7)}

or,
\[ \frac{N \sqrt[3]{Q}}{\Delta h^{1/2}} = \text{const.} = N_s \]  \hspace{1cm} \text{(8)}

When the rotational speed of the impeller \( N \) is (rpm), the volumetric flow rate \( Q \) in (US galpm) and the total head \( \Delta h \) developed by the pump is in (ft), the constant \( N_s \) in equation (8) is known as the specific speed of the pump. The specific speed is used as an index of pump types and always evaluated at the best efficiency point (bep) of the pump. Specific speed vary in the range (400 – 10,000) depends on the impeller type, and has the dimensions of \( \left( \frac{L}{T^2} \right)^{3/4} \). [ British gal=1.2 USgal, 1 ft\(^3\) = 7.48 USgal, 1 m\(^3\) = 264 USgal]

### 4.7.1 Homologous Centrifugal Pumps

Two different size pumps are said to be geometrically similar when the ratios of corresponding dimensions in one pump are equal to those of the other pump. Geometrically similar pumps are said to be homologous. A sets of equations known as the affinity laws govern the performance of homologous centrifugal pumps at various impeller speeds.

For the two homologous pumps, equations (4), and (5) are given
The characteristic performance curves are available for a centrifugal pump operating at a given rotation speed, equations (13), (14), and (15) enable the characteristic performance curves to be plotted for other operating speeds and for other slightly impeller diameters.

Example 4.4
A volute centrifugal pump with an impeller diameter of 0.02 m has the following performance data when pumping water at the best efficiency point (bep). Impeller speed \( N = 58.3 \text{ rev/s} \), capacity \( Q = 0.012 \text{ m}^3/\text{s} \), total head \( \Delta h = 70 \text{ m} \), required \( \text{NPSH} = 18 \text{ m} \), and power = 12,000 W. Evaluate the performance data of an homologous pump with twice the impeller diameter operating at half the impeller speed.
Example 4.5
Calculate the specific speed for these two pumps

**Solution**

Let subscripts 1 and 2 refer to the first and second pump respectively,

\[ \frac{N_1}{N_2} = 2, \quad \frac{D_1}{D_2} = \frac{1}{2} \]

Ratio of capacities

\[ \frac{Q_1}{Q_2} = \left( \frac{N_1}{N_2} \right)^{\frac{2}{3}} \left( \frac{D_1}{D_2} \right)^{\frac{2}{3}} = 2 \times \left( \frac{1}{2} \right)^{\frac{2}{3}} = 1 \]

\[ \Rightarrow \text{Capacity of the second pump } Q_2 = 4 \times Q_1 = 4 \times 0.012 = 0.048 \text{ m}^3/\text{s} \]

Ratio of total heads

\[ \frac{\Delta h_1}{\Delta h_2} = \left( \frac{N_1}{N_2} \right)^{\frac{1}{3}} \left( \frac{D_1}{D_2} \right)^{\frac{1}{3}} = 4 \times \left( \frac{1}{2} \right)^{\frac{1}{3}} = 1 \]

\[ \Rightarrow \text{Total head of the second pump } \Delta h_2 = \Delta h_1 = 70 \text{ m} \]

Ratio of powers

\[ \frac{P_{11}}{P_{12}} = \left( \frac{N_1}{N_2} \right)^{\frac{2}{3}} \left( \frac{D_1}{D_2} \right)^{\frac{2}{3}} = 8 \times \left( \frac{1}{2} \right)^{\frac{2}{3}} = \frac{1}{4} \]

\[ \Rightarrow \text{Assume } \frac{P_{11}}{P_{12}} = \frac{1}{4} \]

\[ \Rightarrow \text{Break power of the second pump } P_{b2} = \frac{1}{4} P_{b1} = \frac{1}{4} \times 12,000 = 48,000 \text{ W} \]

\[ \frac{NPSH_1}{NPSH_2} = \left( \frac{N_1}{N_2} \right)^{\frac{1}{3}} \left( \frac{D_1}{D_2} \right)^{\frac{1}{3}} = 4 \times \left( \frac{1}{2} \right)^{\frac{1}{3}} = 1 \]

\[ \Rightarrow \text{NPSH of the second pump } NPSH_2 = NPSH_1 = 18 \text{ m} \]

4.8 Centrifugal Pumps in Series and in Parallel
4.8.1 Centrifugal Pumps in Parallel

Consider two centrifugal pumps in parallel. The total head for the pump combination ($\Delta h_T$) is the same as the total head for each pump,

$$\Delta h_T = \Delta h_1 = \Delta h_2$$

$$Q_T = Q_1 + Q_2$$

The operating characteristics curves for two pumps in parallel are:

1. Draw $\Delta h$ versus $Q$ for the two pumps and the system.
2. Draw horizontal $\Delta h_T$ line and determine $Q_1$, $Q_2$, and $Q_T$.
3. $Q_T$ (Total) = $Q_1 + Q_2 = Q_5$ (system).
4. If $Q_T \neq Q_5$, repeat steps 2, 3, and 4 until $Q_T = Q_5$.

Another procedure for solution:
1. The same as above.
2. Draw several horizontal lines (4 to 6) for $\Delta h_T$ and determine their $Q_T$.
3. Draw $\Delta h_T$ versus $Q_T$.
4. The duty point is the intersection of $\Delta h_T$ curve with $\Delta h_S$ curve.

4.8.2 Centrifugal Pumps in Series

Consider two centrifugal pumps in series. The total head for the pump combination ($\Delta h_T$) is the sum of the total heads for the two pumps,

$$\Delta h_T = \Delta h_1 + \Delta h_2$$

$$Q_T = Q_1 = Q_2$$

The operating characteristics curves for two pumps in series are:

1. Draw $\Delta h$ versus $Q$ for the two pumps and the system.
2. Draw vertical $Q_T$ line and determine $\Delta h_1$, $\Delta h_2$, and $\Delta h_S$.
3. $Q_T$ (Total) = $Q_1 + Q_2 = Q_5$ (system).
4. If $\Delta h_T \neq \Delta h_5$ repeat steps 2, 3, and 4 until $\Delta h_T = \Delta h_S$.

Another procedure for solution:
1. The same as above.
2. Draw several vertical lines (4 to 6) for $Q_T$ and determine their $\Delta h_T$.
3. Draw $\Delta h_T$ versus $Q_T$.
4. The duty point is the intersection of $\Delta h_T$ curve with $\Delta h_S$ curve.

Exercise
4.1 A centrifugal pump used to take water from reservoir to another through 800 m length and 0.15 m i.d. if the difference in two tank is 8 m, calculate the flow rate of the water and the power required, assume $f=0.004$.

<table>
<thead>
<tr>
<th>$Q \ (m^3/h)$</th>
<th>0</th>
<th>23</th>
<th>26</th>
<th>69</th>
<th>92</th>
<th>115</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta h \ (m)$</td>
<td>17</td>
<td>16</td>
<td>13.5</td>
<td>10.5</td>
<td>6.6</td>
<td>2.0</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0</td>
<td>0.495</td>
<td>0.61</td>
<td>0.63</td>
<td>0.53</td>
<td>0.1</td>
</tr>
</tbody>
</table>

4.2 A centrifugal pump was manufactured to couple directly to a 15 hp electric motor running at 1450 rpm delivering 50 liter/min against a total head 20 m. It is desired to replace the motor by a diesel engine with 1,000 rpm speed and couple it directly to the pump. Find the probable discharge and head developed by the pump. Also find the hp of the engine that would be employed.

4.3 Calculate the available net positive section head NPSH in a pumping system if the liquid density $\rho = 1200 \ kg/m^3$, the liquid dynamic viscosity $\mu = 0.4 \ Pa \ s$, the mean velocity $u = 1 \ m/s$, the static head on the suction side $z = 3 \ m$, the inside pipe diameter $d_i = 0.0526 \ m$, the gravitational acceleration $g = 9.81 \ m/s^2$, and the equivalent length on the suction side ($\sum Le$) = 5.0 m. The liquid is at its normal boiling point. Neglect entrance and exit losses.

4.4 A volute centrifugal pump has the following performance data at the best efficiency point:

- Volumetric flow rate $Q = 0.015 \ m^3/s$
- Total head $\Delta h = 65 \ m$
- Required net positive suction head NPSH = 16 m
- Liquid power $P = 14000 \ W$
- Impeller speed $N = 58.4 \ rev/s$
- Impeller diameter $D = 0.22 \ m$

Evaluate the performance of a homologous pump which operates at an impeller speed of 29.2 rev/s but which develops the same total head $\Delta h$ and requires the same NPSH.
MODULE V
NON -NEUTONIAN FLUID

5.1 Introduction

A Newtonian fluid at a given temperature and pressure has a constant viscosity \( \mu \) which does not depend on the shear rate and, for streamline (laminar) flow, is equal to the ratio of the shear stress (\( \tau \)) to the shear rate (\( \dot{\gamma} = du/dy \)) as shown in equation 5.1:

\[
\mu = \left| \frac{\tau}{du/dy} \right|
\]

5.1

The modulus sign is used because shear stresses within a fluid act in both the positive and negative senses. Gases and simple low molecular weight liquids are all Newtonian, and viscosity may be treated as constant in any flow problem unless there are significant variations of temperature or pressure. Many fluids, including some that are encountered very widely both industrially and domestically, exhibit non-Newtonian behaviour and their apparent viscosities may depend on the rate at which they are sheared and on their previous shear history. At any position and time in the fluid, the apparent viscosity \( \mu_a \) which is defined as the ratio of the shear stress to the shear rate at that point is given by equation 5.2:

\[
\mu_a = \left| \frac{\tau}{du/dy} \right|
\]

5.2

For Newtonian fluids, a plot of shear stress (\( \tau \)), against shear rate (\( \dot{\gamma} = du/dy \)) on Cartesian coordinate is a straight line having a slope equal to the dynamic viscosity (\( \mu \)) but for Non -Newtonian fluid a plot of shear stress against shear rate does not give a straight line.

There are two types of non-Newtonian fluids: (i) Time-independent and, (ii) Time-dependent.

(i) Time-Independent Non-Newtonian Fluids

In this type the apparent viscosity depends only on the rate of shear at any particular moment and not on the time for which the shear rate is applied. For non-Newtonian fluids the relationship between shear stress and shear rate is more complex and represented by equation (5.3 a & b):

\[
\tau = k(-\dot{\gamma})^n \quad (\text{For Power Fluids})
\]

5.3a

\[
\tau = \tau_0 + k(-\dot{\gamma}) \quad (\text{Bingham Plastic Fluid})
\]

5.3b

The shape of the flow curve for time-independent fluids compared with Newtonian fluid is shown in the Figure 5.1, where

A: Newtonian fluids

![Figure 5.1](image-url)
B: Pseudoplastic fluids [power-law n<1] Examples: Polymer solution, detergent.
C: Dilatant fluids [power-law n>1] Examples: Wet beach sand, starch in water.
D: Bingham plastic fluids, it required (τ₀) for initial flow. Examples: Chocolate mixture, soap, sewage sludge, toothpaste.

(ii) Time-Dependent Non-Newtonian Fluids
For this type, the curves of shear stress versus shear rate depend on how long the shear has been active. This type is classified into:
(i) Thixotropic Fluids
Which exhibit a reversible decrease in shear stress and apparent viscosity with time at a constant shear rate. Example Paints.
(ii) Rheopectic Fluids
Which exhibit a reversible increase in shear stress and apparent viscosity with time at a constant shear rate. Example: Gypsum suspensions, bentonite clay.

5.2 Flow Characteristics of Non-Newtonian Fluid
As earlier establish, the velocity distribution for Newtonian fluid of laminar flow through a circular pipe (Figure 5.3), is given by the equation (5.4):
\[ u_r = u_{max} \left[ 1 - \left( \frac{r}{R} \right)^{2} \right] = 2u \left[ 1 - \left( \frac{r}{R} \right)^{2} \right] \]  
5.4
where, \( u_r \): is the mean (average) linear velocity; \( u = \frac{Q}{A} \)
\[ \gamma = \frac{du_r}{dr} = 2u \left( -2 \frac{r}{R} \right) = -4u \frac{r}{R} \]  
5.5
At pipe walls (r = R) \( \gamma = \gamma_w = \frac{du_r}{dr}_{r=R} \)
\[ \gamma_w = -\frac{4u}{R} \Rightarrow \gamma_w = \frac{8u}{d} \]  
5.6
Equation (5.6) is the flow characteristic and therefore, for laminar flow,
\[ \tau_w = -\mu \gamma_w = \mu \frac{8u}{d} \]  
5.7
The force balance on an element of fluid of L length is:
\[ \tau_w \pi dL = \frac{\pi}{4} d^2 \Delta P \]  
5.8
\[ \tau_w = \frac{\Delta P}{4L/d} = \mu \frac{8u}{d} \]  
5.9

5.3 Flow of General Time-Independent Non-Newtonian Fluids
The slope of a log-log plot of shear stress at the pipe walls against flow characteristic \([8u/d]\) at any point along the pipe is the flow behaviour index \((n')\)

\[
n' = \frac{d \ln \tau_w}{d \ln (\frac{8u}{d})} = \frac{d \ln \tau_w}{d \ln (\frac{\Delta P}{4L/d})} = \frac{d \ln \Delta P}{d \ln (\frac{8u}{d})}
\]

Equation (5.10) leads to equation (5.11):

\[
\tau_w = \frac{\Delta P}{4L/d} = Kp \left( \frac{8u}{d} \right)^{n'}
\]

where, \(K_p'\) and \(n'\) are point values for a particular value of the flow characteristic \((8u/d)\). Equation (5.11) can also be written as:

\[
\tau_w = \frac{\Delta P}{4L/d} = Kp \left( \frac{8u}{d} \right)^{n'-1} \frac{8u}{d}
\]

By the analogy of equation (5.9) with equation (5.12), the following equation can be written for non-Newtonian fluids in equations (5.13) to (5.16):

\[
\tau_w = \frac{\Delta P}{4L/d} = (\mu_a)(\frac{8u}{d})
\]

where, \((\mu_a)\) is apparent viscosity for pipe flow.

\[
(\mu_a) = Kp \left( \frac{8u}{d} \right)^{n'-1}
\]

This equation (5.14) gives a point value for the apparent viscosity of non-Newtonian fluid flow through a pipe. Reynolds number for the of non-Newtonian fluids can be written as equation (5.15):

\[
Re = \frac{\rho ud}{(\mu_a)(\frac{8u}{d})} = \frac{\rho ud}{Kp \left( \frac{8u}{d} \right)^{n'-1}}
\]

\[
Re = \frac{\rho \mu^{2+n'}(\frac{8u}{d})}{m}
\]

where, \(m = Kp' (8^{-n'})\). Equations (5.15) or (5.16) gives a point value for \(Re\) at a particular flow characteristic \((8u/d)\).

A point value of the basic friction factor \((\Phi\) or \(J\)) or fanning friction factor \((f')\) for laminar flow can be obtained from equation (5.17)

\[
\Phi = J = 8 / \text{Re} \quad \text{or} \quad f' = 16 / \text{Re}
\]

The pressure drop due to skin friction can be calculated in the same way as for Newtonian fluids:

\[
-\Delta P_{ls} = 4f'(L/d) (pu^2/2)
\]

Equation (5.18) is used for laminar and turbulent flow, and the fanning friction factor \((f')\) for turbulent flow of general time independent non-Newtonian fluids in smooth cylindrical pipes can be calculated from equation (5.19):

\[
f = a/Re^b
\]

where, \(a\) and \(b\) are function of the flow behaviour index \((n')\) as shown in the Table 5.1

**Table 5.1**
There is another equation to calculate \( f' \) for turbulent flow of time-independent non-Newtonian fluids in smooth cylindrical pipes as presented in equation (5.20)

\[
\frac{1}{f'^{1/3}} = \frac{4}{(\eta')^{0.75}} \log \left[ \frac{\text{Re}}{f'^{0.4}} \right] - \frac{0.4}{(\eta')^{1/3}}
\]

5.20

**Example 5.1**

A general time-independent non-Newtonian liquid of density 961 kg/m\(^3\) flows steadily with an average velocity of 1.523 m/s through a tube 3.048 m long with an inside diameter of 0.0762 m. For these conditions, the pipe flow consistency coefficient \( K_p' \) has a value of 1.48 Pa.s\(^{0.3}\) [or 1.48 (kg / m.s\(^2\))s\(^{0.3}\)] and \( n' \) value of 0.3. Calculate the values of the apparent viscosity for pipe flow \( (\mu_a)_p \), the Reynolds number \( \text{Re} \) and the pressure drop across the tube, neglecting end effects.

**Solution**

Apparent viscosity \( (\mu_a)_p \)

\[
K_p' = \frac{8u}{d} = 1.48 \text{ (kg/m)} \text{ s}^{-1.7} [8 \times (1.523)/0.0762]^{-0.7} \text{s}^{-0.7} = 0.04242 \text{ kg/m.s (or Pa.s)}
\]

\[
\text{Re} = \frac{\rho u d}{(\mu_a)_p} = \frac{961 \times (1.523)(0.762)}{0.04242} = 2629
\]

\( f' = a / \text{Re}^b \) from table \( n' = 0.3 \), \( a = 0.0685 \), \( b = 0.325 \)

\[
f' = 0.0685 / 2629^{0.325} = 0.005202
\]

\[
-\Delta P_{fs} = 4f' (L/d)(\rho u^2/2) = 4 \times (0.005202) \times (3.048 / 0.0762) \times [961(1.523)^2/2] = 927.65 \text{ Pa}
\]

**5.4 Flow of Power-Law Fluids in Pipes**

Power-law fluids are those in which the shear stress \( (\tau) \) is related to the shear rate \( (\dot{\gamma}) \) by equation (5.21):

\[
\tau = k(\dot{\gamma})^n
\]

5.21

For shear stress at a pipe wall \( (\tau_w) \) and the shear rate at the pipe wall \( (\dot{\gamma}_w) \), equation (5.21) becomes:

\[
\tau_w = k(\dot{\gamma}_w)^n
\]

5.22

Equation (5.11) gives the relationship between \( (\Delta P) \) and \( (8u/d) \) for general time-independent non-Newtonian fluids. But for power-law fluids the parameters \( K_p' \) and \( n' \) in equation (5.11) are no longer point values but remain constant over a range of \( (8u/d) \), so that for power-law fluids, equation (5.11) can be written as equation (5.23):

\[
\tau_w = \frac{\Delta P}{4L/d} = K_p' \left( \frac{8u}{d} \right)^n
\]

5.23
The shear rate at pipe wall for general time-independent non-Newtonian fluids is:
\[ \dot{\gamma}_w = \frac{8u}{d} \left( \frac{3n' + 1}{4n'} \right) \]
and for power-law fluids
\[ \dot{\gamma}_w = \frac{8u}{d} \left( \frac{3n + 1}{4n} \right) \]

Combine equations (5.23), (5.24), and (5.25) to give the relationship between the general consistency coefficient \((K)\) and the consistency coefficient for pipe flow \((K_p)\).
\[ K_p = \frac{8u}{d} \left( \frac{3n + 1}{4n} \right)^n \]

The apparent viscosity for power-law fluids in pipe flow:
\[ (\mu_a) = K_p \left( \frac{8u}{d} \right)^{n-1} \]

The Reynolds number for non-Newtonian fluids flow in pipe:
\[ \text{Re} = \frac{\rho u d}{(\mu_a)_p} \]

For power-law fluids flow in pipes the \(\text{Re}\) can be written either as:
\[ \text{Re} = \frac{\rho u d}{K_p \left( \frac{8u}{d} \right)^{n-1}} \]
or as:
\[ \text{Re} = \frac{\rho u^{\frac{2-n}{n}} d^n}{m} \]

where, \(m = K_p \left( 8^{n-1} \right) \)

**Example 5.2**
A Power-law liquid of density 961 kg/m³ flows in steady state with an average velocity of 1.523 m/s through a tube 2.67 m length with an inside diameter of 0.0762 m. For a pipe consistency coefficient of 4.46 Pa s^{0.3} [or 4.46 (kg / m s^2)s^{0.3}], calculate the values of the apparent viscosity for pipe flow \((\mu_a)_p\) in Pa.s, the Reynolds number, \(\text{Re}\), and the pressure drop across the tube for power-law indices \(n = 0.3, 0.7, 1.0, \) and 1.5 respectively.

**Solution**
5.5 Friction Losses Due to Form Friction in Laminar Flow

Since non-Newtonian power-law fluids flowing in conduits are often in laminar flow because of their usually high effective viscosity, loss in sudden changes of diameter (velocity) and in fittings are important in laminar flow.

1- Kinetic Energy in Laminar Flow

Average kinetic energy per unit mass = \( u^2 / 2 \alpha \) \( \text{[m}^2/\text{s}^2 \text{ or J/kg]} \)

\[ \alpha = \frac{(2n+1)(5n+3)}{3(3n+1)^2} \quad \text{in laminar flow} \]

- For Newtonian fluids \( (n = 1.0) \) \( \Rightarrow \alpha = 1/2 \) in laminar flow
- For power-law non-Newtonian fluids \( (n < 1.0 \text{ or } n > 1.0) \)

2- Losses in Contraction and Fittings

The frictional pressure losses for non-Newtonian fluids are very similar to those for Newtonian fluids at the same generalized Reynolds number in laminar and turbulent flow for contractions and also for fittings and valves.

3- Losses in Sudden Expansion

For a non-Newtonian power-law fluid flow in laminar flow through a sudden expansion from a smaller inside diameter \( d_1 \) to a larger inside diameter \( d_2 \) of circular cross-sectional area, then the energy losses is

\[ F_c = \frac{n+3}{2(5n+3)} \left( \frac{d_1}{d_2} \right)^4 - \left( \frac{d_1}{d_2} \right)^{4 + \frac{3(3n+1)}{2(5n+3)}} \frac{3n+1}{2n+1} u_i^2 \]

5.6 Turbulent Flow and Generalized Friction Factor
The generalized Reynolds number has been defined as

\[ Re = \frac{d^n u^{2-n} \rho}{m} \]

where, \( m = K p^{0.8} = K 8^{n-1} (3n+1/4) \)

The fanning friction factor is plotted versus the generalized Reynolds number. Since many non-Newtonian power-law fluids have high effective viscosities, they are often in laminar flow. The correction for smooth tube also holds for a rough pipe in laminar flow.

**Figure 5.4 : Friction factor chart for purely viscous non-Newtonian fluids**

For rough pipes with various values of roughness ratio (e/d), this figure cannot be used for turbulent flow, since it is derived for smooth pipes.

**Example 5.3**

A pseudoplastic fluid that follows the power-law, having a density of 961 kg/m³ is flowing in steady state through a smooth circular tube having an inside diameter of 0.0508 m at an average velocity of 6.1 m/s. the flow properties of the fluid are \( n' = 0.3, K p = 2.744 \text{ Pa.s}^{0.7} \). Calculate the frictional pressure drop across the tubing of 30.5 m long.

**Solution**

\[ Re = \frac{d^n u^{2-n} \rho}{m} = (1.523)^{0.3} (6.1)^{1.7} (961) / 2.744 (8)^{0.7} \]

\[ = 1.328 \times 10^4 \]

From Figure for \( Re = 1.328 \times 10^4, n' = 0.3 \Rightarrow f = 0.0032 \)
5.1 The shear stress in power-law liquids in steady state laminar flow is given by the equation
\[ \tau_s = K \left( \frac{du_s}{dr} \right)^n, \]
show that the velocity distribution is given by the following equation
\[ u_s = u_{\text{max}} \left( 1 - \left( \frac{r}{R} \right)^{n+1} \right), \]
where \( u_{\text{max}} = \frac{n}{n+1} \left( \frac{\Delta P_{fs}}{2KL} \right)^\frac{1}{n} R^{n+1} \).

**Hint:** \( \tau_s (2\pi r L) = -\Delta P_{fs} (\pi r^2) \Rightarrow \tau_s = -\frac{\Delta P_{fs}}{2L} r \)

5.2 Calculate the frictional pressure gradient \( -\Delta P_{fs}/L \) for a time independent non-Newtonian fluid in steady state flow in a cylindrical tube if
- the liquid density \( \rho = 1000 \text{ kg/m}^3 \)
- inside diameter of the tube \( d = 0.08 \text{ m} \)
- the mean velocity \( u = 1.0 \text{ m/s} \)
- the point pipe consistency coefficient \( K' = 2 \text{ Pa.s}^{0.5} \)
- and the flow behaviour index \( n' = 0.5 \).

\[ \Delta P_{fs} = 4f \left( \frac{L}{d} \right) \left( \frac{p u^2}{2} \right) = 4(0.0032) (30.5 / 0.0508)[961(6.1)^2/2] \]

\[ \Rightarrow \Delta P_{fs} = 134.4 \text{ kPa} \]
6.1 Introduction
It is important to be able to measure and control the amount of material entering and leaving a chemical and other processing plants. Since many of the materials are in the form of fluids, they are flowing in pipes or conduits. Different types of devices are used to measure the flow of fluids. The flow of fluids is most commonly measured using head flow meters. The operation of these flow meters is based on the Bernoulli’s equation. The most important class of flow meter is that in which the fluid is either accelerated or retarded at the measuring section by reducing the flow area, and the change in the kinetic energy is measured by recording the pressure difference produced.

6.2 Flow Measurement Apparatus

Head flow meters include orifice, venture meter, flow nozzles, Pitot tubes, and wiers. They consist of primary element, which causes the pressure or head loss and a secondary element, which measures it.

6.2.1 Pitot Tube

The Pitot tube is used to measure the local velocity at a given point in the flow stream and not the average velocity in the pipe or conduit. In the Figures 6.1, a sketch of this simple device is shown. One tube, the impact tube, has its opening normal to the direction of flow and the static tube has its opening parallel to the direction of flow. In the pilot tube, a small element of fluid is brought to rest at an orifice situated at right angles to the direction of flow. The flow rate is then obtained from the difference between the impact and the static pressure. With this instrument the velocity measured is that of a small filament of fluid.

Point 2 called stagnation point at which the impact pressure is \( u_2 \).
The fluid flows into the opening at point 2, pressure builds up, and then remains stationary at this point, called “Stagnation Point”. The difference in the stagnation pressure (impact pressure) at this point (2) and the static pressure measured by the static tube represents the pressure rise associated with the direction of the fluid.

**Impact pressure head = Static pressure head + kinetic energy head**

Since Bernoulli’s equation is used for ideal fluids, therefore for real fluids the last equations of local velocity become:

\[
\begin{align*}
\frac{P_1}{\rho g} + \frac{u_1^2}{2\alpha_1 g} + z_1 &= \frac{P_2}{\rho g} + \frac{u_2^2}{2\alpha_2 g} + z_2 \\
\Rightarrow u_2 &= \sqrt{\frac{2(P_2 - P_1)}{\rho}} - \sqrt{2\rho g h} = \sqrt{\frac{2R(\rho_m - \rho)g}{\rho}}
\end{align*}
\]

where, \(\Delta P = R(\rho_m - \rho)g\)

The fluid flows into the opening at point 2, pressure builds up, and then remains stationary at this point, called “Stagnation Point”. The difference in the stagnation pressure (impact pressure) at this point (2) and the static pressure measured by the static tube represents the pressure rise associated with the direction of the fluid.

The first method: the velocity is measured at the exact centre of the tube to obtain \(u_{max}\) then by using the Figure 6.1, the average velocity can be obtained.

The second method: readings are taken at several known positions in the pipe cross section and then a graphical or numerical integration is performed to obtain the average velocity, from the following equation;

\[
u = \frac{\int_{A} u \cdot dA}{A}
\]

**Example 6.1**

Find the local velocity of the flow of an oil of sp.gr. = 0.8 through a pipe, when the difference of mercury level in differential U-tube manometer connected to the two tapping of the Pitot tube is 10 cm Hg. Take \(C_p = 0.98\).

**Solution:**

\[
u_2 = C_p \sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} = 0.98 \sqrt{\frac{2(0.1)(13600 - 800)9.81}{800}} = 5.49 \text{ m/s}
\]

**Example 6.2**

A Pitot tube is placed at a centre of a 30 cm I.D. pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.84 of the centre velocity (i.e. \(u/u_x = 0.94\)). Find the discharge through the pipe if:

i- The fluid flow through the pipe is water and the pressure difference between orifice is 6 cm H₂O.

ii- The fluid flow through the pipe is oil of sp.gr. = 0.78 and the reading manometer is 6 cm H₂O.

Take \(C_p = 0.98\).
Solution:

i- \( u_x = \frac{C_P}{2g}\Delta h = 0.98\sqrt{2(9.81)(0.06)} = 1.063 \text{ m/s} \)

\( u = 0.84 (1.063) = 0.893 \text{ m/s} \), \( Q = \frac{A.u}{\pi/4(0.3)^2} (0.893) = 0.063 \text{ m}^3/\text{s} \)

ii- \( u_x = \frac{C_P}{\rho} \frac{2R(\rho_a - \rho_\infty)}{\rho g} = 0.98 \frac{2(0.06)(13600 - 780)(9.81)}{780} = 0.565 \text{ m/s} \)

\( u = 0.84 (0.565) = 0.475 \text{ m/s} \), \( Q = \frac{A.u}{\pi/4(0.3)^2} (0.475) = 0.0335 \text{ m}^3/\text{s} \)

Example 6.3
A Pitot tube is inserted in the pipe of 30 cm I.D. The static pressure head is 10 cm Hg vacuum, and the stagnation pressure at centre of the pipe is 0.981 N/cm gauge. Calculate the discharge of water through the pipe if \( u/u_{\text{max}} = 0.85 \). Take \( C_P = 0.98 \).

Solution:

\( P_1 = -10 \text{ cm Hg} (13600) 9.81 (\text{m} / 100 \text{ cm}) = -13.3416 \text{ kPa} \)

\( P_2 = 0.981 \text{ N/cm}^2 (\text{m} / 100 \text{ cm})^2 = 9.81 \text{ kPa} \)

\( \Delta P = P_2 - P_1 = 9.81 - (-13.3416) = 23.1516 \text{ kPa} \)

\( u_x = \frac{C_P}{\rho} \frac{2(-\Delta P)}{\rho g} = 0.98 \frac{2(23.1516 \times 10^3)}{1000} = 6.67 \text{ m/s} \)

\( u = 0.85 (6.67) = 5.67 \text{ m/s} \), \( Q = \frac{A.u}{\pi/4(0.3)^2} (5.67) = 0.4 \text{ m}^3/\text{s} \)

Example 6.4
A Pitot tube is used to measure the air flow rate in a circular duct 60 cm I.D. The flowing air temperature is 65.5ºC. The Pitot tube is placed at the centre of the duct and the reading \( R \) on the manometer is 10.7 mm of water. A static pressure measurement obtained at the Pitot tube position is 205 mm of water above atmospheric. Take \( C_P = 0.98, \mu = 2.03 \times 10^{-5} \text{ Pa.s} \).

a. Calculate the velocity at the centre and the average velocity.

b. Calculate the volumetric flow rate of the flowing air in the duct.

Solution:

a-

\( P_i \) = the static pressure

\( P_i \) (gauge) = 0.205 (1000) 9.81 = 2011 kPa

\( P_i \) (abs) = 2011 + 1.01325 \times 10^5 \text{ Pa} = 1.03336 \times 10^5 \text{ Pa} 

\( \rho_{\text{air}} = \text{Mwt. P}/(R.T) = 29 (1.03336 \times 10^5) / [(8314 \text{ Pa} \cdot \text{m}^3/\text{kmol.K}) (65.5 + 273.15)] \)

\( = 1.064 \text{ kg/m}^3 \)

\( u_x = \frac{C_P}{\rho} \frac{2R(\rho_a - \rho_\infty)}{\rho g} = 0.98 \frac{2(0.0107)(1000 - 1.064)(9.81)}{1.064} = 14.04 \text{ m/s} = u_{\text{max}} \)

\( Re_{\text{max}} = \rho_{\text{max}} d/\mu = 1.064(14.04)(0.6)/2.03 \times 10^{-5} = 4.415 \times 10^5 \)

From Figure \( u/u_{\text{max}} = 0.85 \Rightarrow u = 0.85 (14.04) = 11.934 \)

b-

\( Q = \frac{A.u}{\pi/4(0.6)^2} (11.934) = 3.374 \text{ m}^3/\text{s} \)

6.2.2 Measurement by Flow Through a Constriction
In measuring devices where the fluid is accelerated by causing it to flow through a constriction, the kinetic energy is thereby increased and the pressure energy therefore decreases. The flow rate is obtained by measuring the pressure difference between the inlet of the meter and a point of reduced pressure. **Venturi meters, orifice meters, and flow nozzles** measure the volumetric flow rate \( Q \) or average (mean linear) velocity \( u \). In contrast the Pitot tube measures a point (local) velocity \( u_x \).

### 6.2.2.1 Venturi Meter

Venturi meters consist of three sections as shown in Figure 6.2:

![Figure 6.3: Venturi Meter](image)

- From continuity equation \( A_1 u_1 = A_2 u_2 \Rightarrow u_1 = (A_2/A_1) u_2 \)
- From Bernoulli’s equation between points 1 and 2

\[
\frac{P_1}{\rho g} + \frac{u_1^2}{2g} + \phi_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + \phi_2
\]

\[
\Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{u_1^2 - u_2^2}{2g} = \frac{u_1^2}{2g} \left[ 1 - \frac{A_2}{A_1} \right] = \frac{u_2^2}{2g} \left[ \frac{A_1}{A_2} \right]
\]

\( \Rightarrow \quad u_2 = \sqrt{\frac{2(-\Delta P)}{\rho} \left[ 1 - \frac{A_2}{A_1} \right]} = \sqrt{2g \Delta h} \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \)

or \( \quad u_2 = \sqrt{2g \Delta h \left[ 1 - \frac{A_1}{A_2} \right]} \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \)

or \( \quad u_2 = \sqrt{2R(\rho_m - \rho)g} \left[ 1 - \frac{A_1}{A_2} \right] \frac{A_1}{\rho \sqrt{A_1^2 - A_2^2}} \)

All these equation of velocity at throat \( u_2 \), which derived from Bernoulli’s equation are for ideal fluids. Using a coefficient of discharge \( C_d \) to take account of the frictional losses in the meter and of the parameters of kinetic energy correction \( \alpha_1 \) and \( \alpha_2 \). Thus the volumetric flow rate will be obtained by:
Example 6.5
A horizontal Venturi meter with \( d_1 = 20 \text{ cm} \), and \( d_2 = 10 \text{ cm} \), is used to measure the flow rate of oil of sp.gr. = 0.8, the discharge through venture meter is 60 lit/s. Find the reading of (oil-Hg) differential. Take \( C_d = 0.98 \).

Solution:
\[
Q = \frac{u_2 A_2}{C_d} = \frac{2(-\Delta P)}{\rho} \left[ \frac{A_2}{A_1} \right] = \frac{2(-\Delta P)}{\rho} \frac{A_1 A_2 \sqrt{A_1^2 - A_2^2}}{A_1^2 - A_2^2}
\]

or
\[
Q = C_d \sqrt{2g\Delta h \frac{A_2^2}{1 - (A_1 / A_2)^2}} = C_d \sqrt{2g\Delta h} \frac{A_1 A_2 \sqrt{A_1^2 - A_2^2}}{A_1^2 - A_2^2}
\]

or
\[
Q = C_d \sqrt{2R(\rho_m - \rho)g \frac{A_2^2}{1 - (A_1 / A_2)^2}} = C_d \sqrt{2R(\rho_m - \rho)g} \frac{A_1 A_2 \sqrt{A_1^2 - A_2^2}}{A_1^2 - A_2^2}
\]

For many meters and for \( \text{Re} > 10^4 \) at point 1,
\[
C_d = 0.98 \quad \text{for} \quad d_1 < 20 \text{ cm}
\]
\[
C_d = 0.99 \quad \text{for} \quad d_1 > 20 \text{ cm}
\]

Example 6.6
A horizontal Venturi meter is used to measure the flow rate of water through the piping system of 20 cm I.D, where the diameter of throat in the meter is \( d_2 = 10 \text{ cm} \). The pressure at inlet is 17.658 N/cm\(^2\) gauge and the vacuum pressure of 35 cm Hg at throat. Find the discharge of water. Take \( C_d = 0.98 \).

Solution:
\[
P_1 = 17.658 \text{ N/cm}^2 \quad (100 \text{ cm} / \text{m})^2 = 176580 \text{ Pa}
\]
\[
P_2 = -35 \text{ mm Hg} \quad \text{(m} / \text{100 cm}) = -46695.6 \text{ Pa}
\]
\[
P_1 - P_2 = 176580 - (-46695.6) = 223275.6 \text{ Pa}
\]
\[
Q = \frac{u_2 A_2}{C_d} = \frac{2(-\Delta P)}{\rho} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = 0.98 \sqrt{\frac{2(223275.6)(0.2)^2[(\pi/4)(0.1)]}{1000(0.2)^2 - (0.1)^2}}
\]

\[
\Rightarrow Q = 0.168 \text{ m}^3/\text{s}
\]

Example 6.7
A Venturi meter is to be fitted to a 25 cm diameter pipe, in which the maximum flow is 7200 lit/min and the pressure head is 6 m of water. What is the maximum diameter of throat, so that there is non-negative head on it?
**Solution:**
\[ h_1 = 6 \text{ m H}_2\text{O} \]
Since the pressure head at the throat is not to be negative, or maximum it can be zero (i.e. \( h_2 = 0 \)). Therefore;
\[ \Delta h = h_1 - h_2 = 6 - 0 = 6 \text{ m H}_2\text{O} \]
\[ Q = u_2 A_2 = 7200 \text{ lit/min} (\text{m}^3/1000\text{lit}) (\text{min} / 60 \text{ s}) = 0.12 \text{ m}^3/\text{s} \]
\[ \Rightarrow 0.12 = C_d \sqrt{2g\Delta h} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = 1.0 \sqrt{2(9.81)\frac{(0.25)^2}{(\pi/4)(d_2)^2}} \sqrt{(0.25)^4 - (d_1)^4} \]
\[ 0.225 = \frac{d_2^4}{\sqrt{(0.25)^4 - (d_1)^4}} \Rightarrow 0.0507 = \frac{d_2^4}{(0.25)^4 - d_1^4} \]
\[ \Rightarrow d_1^4 + 0.507d_1^4 - 1.983 \times 10^{-4} = 0 \quad \Rightarrow d_1^4 = 1.887237 \times 10^{-4} \]
\[ \Rightarrow d_2 = 0.1172 \text{ m} = 11.72 \text{ cm} \]

**Note:** In case of using **vertical** or **inclined** Venturi meter instead of horizontal one, the same equations for estimation of the actual velocity are used.

**Example 6.8**
A (30cm x 15cm) Venturi meter is provided in a vertical pipe-line carrying oil of sp.gr. = 0.9. The flow being upwards and the difference in elevations of throat section and entrance section of the venture meter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Take \( C_d = 0.98 \) and calculate: (i) The discharge of oil (ii)- The pressure difference between the entrance and throat sections.

**Solution:**
\[ i- \quad Q = u_1 A_1 = C_d \sqrt{\frac{2R(\rho_u - \rho)g}{\rho}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = 0.98 \sqrt{\frac{2(0.25)(12700)9.81}{900}} \left[ \frac{0.3^2 (\pi/4)(0.15)^2}{\sqrt{0.3^4 - 0.15^4}} \right] \]
\[ = 0.1488 \text{ m}^3/\text{s} \]
\[ ii- \quad \text{Applying Bernoulli’s equation at points 1 and 2} \]
\[ \frac{P_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + z_2 \]
\[ \frac{P_1 - P_2}{\rho g} = z_2 + \frac{u_2^2 - u_1^2}{2g} \]
\[ u_1 = 0.1488/(\pi/4 0.3^2) = 2.1 \text{ m/s}, \quad u_2 = 0.1488/(\pi/4 0.15^2) = 8.42 \text{ m/s} \]
\[ \Rightarrow P_1 - P_2 = 900 (9.81) \left[ 0.3 + (8.42^2 - 2.1^2)/2(9.81) \right] \]
\[ = 32.5675 \text{ kPa} \]
but \( P_1 - P_2 = 0.25 (13600-900)(9.81) = 31.1467 \text{ kPa} \]
\[ \% \text{ error} = 4.36 \% \]

**6.2.2.2 Orifice Meter**
The primary element of an orifice meter is simply a flat plate containing a drilled located in a pipe perpendicular to the direction of fluid flow as shown in Figure 6.4.

![Figure 6.4: Orifice Meter](image)

At point 2 in the pipe the fluid attains its maximum mean linear velocity $u_2$ and its smallest cross-sectional flow area $A_2$. This point is known as "the vena contracta". It occurs at about one-half to two pipe diameters downstream from the orifice plate. Because of relatively large friction losses from the eddies generated by the expanding jet below vena contracta, the pressure recovery in orifice meter is poor.

Using a coefficient of discharge $C_d$ to take into account the frictional losses in the meter and of parameters $C_c$, $\alpha_1$, and $\alpha_2$. Thus the velocity at orifice or the discharge through the meter is:

$$Q = C_d \sqrt{\frac{2(-\Delta P)}{\rho} \left[ \frac{A_2^2}{1-(A_2/A_1)^2} \right]} = C_d \sqrt{\frac{2(-\Delta P)}{\rho} \frac{A_1 A_s}{\sqrt{A_1^2 - A_2^2}}}$$

or

$$Q = C_d \sqrt{2g\Delta h \left[ \frac{A_2^2}{1-(A_2/A_1)^2} \right]} = C_d \sqrt{2g\Delta h \frac{A_1 A_s}{\sqrt{A_1^2 - A_2^2}}}$$

or

$$Q = C_d \sqrt{\frac{2R(\rho_a - \rho)g}{\rho} \left[ \frac{A_2^2}{1-(A_2/A_1)^2} \right]} = C_d \sqrt{\frac{2R(\rho_a - \rho)g}{\rho} \frac{A_1 A_s}{\sqrt{A_1^2 - A_2^2}}}$$
Figure 6.5: The discharge coefficient for orifice meter.

The holes in orifice plates may be *concentric, eccentric or segmental* as shown in Figure 6.6. Orifice plates are prone to damage by erosion.

![Concentric, eccentric and segmental orifice plates](image)

**Figure 6.6: Concentric, eccentric and segmental orifice plates**

**Example 6.9**

An orifice meter consisting of 10 cm diameter orifice in a 25 cm diameter pipe has $C_d = 0.65$. The pipe delivers oil of sp.gr. = 0.8. The pressure difference on the two sides of the orifice plate is measured by mercury oil differential manometer. If the differential gauge is 80 cm Hg, find the rate of flow.
Example 6.10
Water flow through an orifice meter of 25 mm diameter situated in a 75 mm diameter pipe at a rate of 300 cm$^3$/s, what will be the difference in pressure head across the meter $\mu = 1.0$ mPa.s.

**Solution:**

$Q = C_d \sqrt{2g \Delta h \frac{A_1 - A_2}{\sqrt{A_1^2 - A_2^2}}} = 0.65 \sqrt{\frac{2(0.8)(13600 - 800)9.81}{800} \left(\frac{\pi}{4}(0.1)^2(0.25)^2\right)}$

$\Rightarrow Q = 0.08196 \text{ m}^3/\text{s}$

Example 6.11
Water flow at between 3000-4000 cm$^3$/s through a 75 mm diameter pipe and is metered by means of an orifice. Suggest a suitable size of orifice if the pressure difference is to be measured with a simple water manometer. What approximately is the pressure difference recorded at the maximum flow rate? $C_d = 0.6$.

**Solution:**

The largest practicable height of a water manometer is 1.0 m

$Q = C_d \sqrt{2g \Delta h \frac{A_1 - A_2}{\sqrt{A_1^2 - A_2^2}}}$

The maximum flow rate = $4 \times 10^3 \text{ m}^3/\text{s}$

$4 \times 10^3 \text{ m}^3/\text{s} = 0.6 \sqrt{2(9.81)(1.0)} \left(\frac{\pi}{4}(0.05)^2(d_o)^2\right) \left[\left(\frac{0.05}{d_o}\right)^4 - (d_o)^4\right]$

$\Rightarrow \frac{d_o^2}{\sqrt{(0.05)^4 - d_o^4}} = 0.7665 \Rightarrow d_o^2 = 3.67 \times 10^{-6} - 0.5875d_o^4$

$\Rightarrow d_o = 0.039 \text{ m} = 39 \text{ mm}$

$(P_1 - P_2) = \Delta h \rho g = 1.0 (1000)(9.81) = 9810 \text{ Pa}$

6.2.2.3 The Nozzle

The nozzle is similar to the orifice meter other than that it has a *converging tube* in place of the orifice plate, as shown in Figure 6.7. The velocity of the fluid is gradually increased and the contours are so designed that almost frictionless flow takes place in the converging portion; the outlet corresponds to the *vena contracta* on the orifice meter. The nozzle has a constant high coefficient of discharge (ca.
over a wide range of conditions because the coefficient of contraction is unity, though because the simple nozzle is not fitted with a diverging cone, the head lost is very nearly the same as with an orifice. Although much more costly than the orifice meter, it is extensively used for metering steam. When the ratio of the pressure at the nozzle exit to the upstream pressure is less than the critical pressure ratio $\omega$, the flow rate is independent of the downstream pressure and can be calculated from the upstream pressure alone.

![Diagram of nozzle](image)

Figure 6.7: Figures of nozzle (a) General arrangement (b) Standard nozzle ($A_o/A_1$) is less than 0.45. Left half shows construction for corner tappings. Right half shows construction for piezometer ring (c) Standard nozzle where ($A_o/A_1$) is greater than 0.45

6.2.3 Variable Area Meters - Rotameters

In the previous flow meters, the area of constriction or orifice is constant, and the pressure drop is dependent on the rate of the flow (due to conversions between the pressure energy with kinetic energy). In the Rotameter the drop in pressure is constant and the flow rate is a function of the area of constriction. When the fluid is flowing, the float rises until its weight is balanced by the up thrust of the fluid. Its position then indicates the rate of flow.
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\[ (\Delta P) \] is the pressure difference over the float, \((\Delta P) = P_1 - P_2\)

The area of flow is the annulus formed between the float and the wall of the tube. This meter may thus be considered, as an orifice meter with a variable aperture, and the equation of flow rate already derived are therefore applicable with only minor changes.

\[
Q = \frac{C_d}{\rho} \sqrt{\frac{2(\Delta P)}{\rho}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}
\]

\[
Q = \frac{C_d}{\rho} \sqrt{\frac{2V_f g (\rho_f - \rho)}{\rho A_f}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}
\]

where,
- \(A_1\) : cross-section area of the tube when the float arrived.
- \(A_2\) : cross-section area of the annulus (flow area).

**Example 6.12**
A rotameter tube of 0.3 m long with an internal diameter of 25 mm at the top and 20 mm at the bottom. The diameter of float is 20 mm, its sp.gr. is 4.8 and its volume is 6 cm\(^3\). If the coefficient of discharge is 0.7, what will be the flow rate water when the float is half way up the tube?
Example 6.13

A rotameter has a tube of 0.3 m long, which has an internal diameter of 25 mm at the top and 20 mm at the bottom. The diameter of float is 20 mm, its effective sp.gr. is 4.8 and its volume is 6.6 cm$^3$. If the coefficient of discharge is 0.72, what height will the float be when metering water at 100 cm$^3$/s?

**Solution:**

$$A_1 = \pi/4 d_1^2, \quad d_1 = d_f + 2x$$

To find $x$

1. $0.25/30 = x/15$ \Rightarrow $x = 0.125$ cm
2. $\tan(\theta) = 0.25 / 30 = x/15$ \Rightarrow $x = 0.125$ cm

$\Rightarrow d_1 = 2 + 2(0.125) = 2.25$ cm

$\Rightarrow A_1 = \pi/4 (0.0225)^2 = 3.976 \times 10^{-4} \text{ m}^2$

$A_2 = A_1 - A_f = 3.976 \times 10^{-4} - \pi/4 (0.02)^2$

$= 8.345 \times 10^{-5} \text{ m}^2$

$$Q = C_d \sqrt{ \frac{2V_f g (\rho_f - \rho)}{\rho_f A_f} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} }$$

$$= 0.7 \sqrt{ \frac{26 \times 10^{-4}(9.81)(4.8 - 1)}{\pi/4(0.02)^2} \frac{(3.976 \times 10^{-4})(8.345 \times 10^{-5})}{\sqrt{(3.976 \times 10^{-4})^2 - (8.345 \times 10^{-5})^2}} }$$

$= 7.13 \times 10^{-3} \text{ m}^3/\text{s}$
6.2.4 The Notch or Weir

The flow of liquid presenting a free surface (open channels) can be measured by means of a weir. The pressure energy converted into kinetic energy as it flows over the weir, which may or may not cover the full width of the stream, and a calming screen may be fitted before the weir. Then the height of the weir crest gives a measure of the rate of flow. The velocity with which the liquid leaves depends on its initial depth below the surface.

Many shapes of notch are available of which three shapes are given here as shown in Figure 6.8.

![Figure 6.8: The Weir](image)

6.2.4.1 Rectangular Notch

To prove this equation applies Bernoulli’s equation between points M and N as shown in Figure:

\[
\frac{P_M}{\rho g} + \frac{u_M^2}{2g} + z_M = \frac{P_N}{\rho g} + \frac{u_N^2}{2g} + z_N
\]

The cross sectional area of flow at point M is larger than that at notch (point N), then \(u_M \approx 0\)

\[P_M = P_N = P_o\text{ atmospheric pressure}\]

\[
z_M - z_N = \frac{u_N^2}{2g}
\]

\[\therefore u_N = \sqrt{2gh}\]
Example 6.14: A rectangular notch 2.5 m wide has a constant head of 40 cm, find the discharge over the notch where $C_d = 0.62$

**Solution:**

$$Q = \frac{2}{3} C_d b \sqrt{2gH^{3/2}} = 2/3 \times 0.62 \times (2.5) \times (2 \times 9.81)^{0.5} \times (0.4)^{3/2}$$

$$Q = 1.16 \text{ m}^3/\text{s}$$

Example 6.15: A rectangular notch has a discharge of $21.5 \text{ m}^3/\text{min}$, when the head of water is half the length of the notch. Find the length of the notch where $C_d = 0.6$.

**Solution:**

$$Q = \frac{2}{3} C_d b \sqrt{2gH^{3/2}} \Rightarrow 21.5/60 = 2/3 \times 0.6 \times (b) \times (2 \times 9.81)^{0.5} \times (0.5 \times b)^{3/2}$$

$$\Rightarrow b^{5/2} = 0.572 \Rightarrow b = (0.572)^{2/5} = 0.8 \text{ m}$$

6.2.4.2 Triangular Notch

A triangular notch is also called a $V$-notch.

- $H$: height of liquid above base of the apex of the notch.
- $\theta$: Angle of the notch.

$$\tan (\theta/2) = x / H = x' / (H-h)$$

The width of the notch at liquid surface = $2x = 2H \tan(\theta/2)$

The width of the strip = $2x' = 2(H-h) \tan(\theta/2)$

The area of the strip = $2x' \, dh = 2(H-h) \tan(\theta/2) \, dh$

The theoretical velocity of water through the strip = $\sqrt{2gh}$

The discharge over the notch $dQ = \frac{2}{15} \frac{C_d \tan(\theta/2) \sqrt{2gH^{3/2}}}{\sqrt{2gH^{3/2}}}$

$$Q = \frac{8}{15} C_d \tan(\theta/2) \sqrt{2gH^{5/2}}$$

If $C_d = 0.6$ and $\theta = 90^\circ \Rightarrow Q = 1.417 H^{5/2}$
Example 6.16: During an experiment in a laboratory, 50 liters of water flowing over a right-angled notch was collected in one minute. If the head of still is 50mm. Calculate the coefficient of discharge of the notch.

**Solution:**
\[ Q = \frac{8}{15} C_d \tan(\theta/2) \sqrt{2gH^{3/2}} = 50 \text{ lit/min} \left( \frac{m^3}{1000 \text{lit}} \right) \left( \frac{\text{min}}{60 \text{s}} \right) = 8.334 \times 10^{-4} \text{ m}^3/\text{s} \]
\[ \Rightarrow C_d = \frac{8.334 \times 10^{-4}}{\left[ \frac{8}{15} \times 2 \times 9.81 \right]^{0.5} \tan(\theta/2) \left( 0.05 \right)^{3/2}} = 0.63 \]

Example 6.18:
A rectangular channel 1.5 m wide is used to carry 0.2 m³/s water. The rate of flow is measured by placing a 90° V-notch weir. If the maximum depth of water is not to exceed 1.2 m, find the position of the apex of the notch from the bed of channel. \( C_d = 0.6 \).

**Solution:**
\[ Q = 1.417 H^{3/2} \Rightarrow H^{3/2} = \frac{(0.2 \text{ m}^3/\text{s})}{1.417} \Rightarrow H = 0.46 \text{ m} \]
The maximum depth of water in channel = 1.2 m

H is the height of water above the apex of notch.

Apex of triangular notch is to be kept at distance = 1.2 - 0.46 = 0.74 m from the bed of channel.

6.2.4.3 Trapezoidal Notch
A trapezoidal notch is a combination of a rectangular notch and triangular notch as shown in Figure;

![Image](image.png)

Discharge over the trapezoidal notch,
\[ Q = \left[ \text{Discharge over the rectangular notch} + \text{Discharge over the triangular notch} \right] \]
\[ Q = \frac{2}{3} C_{d1} b \sqrt{2gH^{3/2}} + \frac{8}{15} C_{d2} \tan(\theta/2) \sqrt{2gH^{3/2}} \]

Example 6.18:
A trapezoidal notch 120 cm wide at top and 45 cm at the bottom has 30 cm height. Find the discharge through the notch, if the head of water is 22.5 cm. \( C_{d1} = C_{d2} = 0.6 \).

**Solution:**
\[ x = \frac{(120 + 45)}{2} = 37.5 \text{ cm} \]
\[ \tan(\theta/2) = \frac{x}{30} = 37.5/30 = 1.25 \]
\[ Q = \frac{2}{3} C_{d1} b \sqrt{2gH^{3/2}} + \frac{8}{15} C_{d2} \tan(\theta/2) \sqrt{2gH^{3/2}} \]
\[ = 2/3 \times 0.6 \times (0.45) \times (2 \times 9.81)^{0.5} \times (0.225)^{3/2} + 8/15 \times 1.25 \times (2 \times 9.81)^{0.5} \times (0.225)^{3/2} \]
\[ = 0.1276 \text{ m}^3/\text{s} \]

6.3 Unsteady State Problems
Example 6.19
A reservoir 100 m long and 100 m wide is provided with a rectangular notch 2 m long. Find the time required to lower the water level in the reservoir from 2 m to 1 m. \(C_d = 0.6\).

**Solution:**
Let, at some instant, the height of the water above the base of the notch be \(h\) and the liquid level fall to small height \(dh\) in time \(dt\).

The volume of water discharged in time \(dt\) is:

\[
\frac{dV}{dt} = -A \frac{2}{3} C_d b \sqrt{2g} h^{5/2} = -A \frac{dh}{dt}
\]

\[
\Rightarrow \int_0^T dt = \frac{-A}{(2/3)C_d b \sqrt{2g}} \int_{H_1}^{H_2} h^{3/2} dh
\]

\[
T = \frac{3}{2} C_d b \sqrt{2g} \left[ \frac{1}{H_2^{1/2}} - \frac{1}{H_1^{1/2}} \right]
\]

\[
= \frac{3A}{C_d b \sqrt{2g}} \left[ \frac{1}{H_2^{1/2}} - \frac{1}{H_1^{1/2}} \right] = \frac{3 \times 10^4}{0.6(2) \sqrt{2} \times 9.81 (\sqrt{1} - \sqrt{2})} = 1653.1 \text{ sec} = 27 \text{ min}, 33 \text{ sec}
\]

Example 6.20
A tank 25 m long and 15 m wide is provided with a right-angled V-notch. Find the time required to lower the level in the tank from 1.5 m to 0.5 m. \(C_d = 0.62\).

**Solution:**
Let, at some instant, the height of the liquid above the apex of the notch be \(h\) and a small volume of the liquid \(dv\) flow over the notch in a small interval of time \(dt\), reducing the liquid level by an amount \(dh\) in the tank.

\[
dV = -A \frac{8}{15} C_d \sqrt{2g \tan(\theta/2)} h^{5/2} = -A \frac{dh}{dt}
\]

\[
\Rightarrow \int_0^T dt = \frac{-A}{(8/15)C_d \sqrt{2g \tan(\theta/2)}} \int_{H_1}^{H_2} h^{3/2} dh
\]

\[
T = \frac{15}{8} C_d \sqrt{2g \tan(\theta/2)} \left[ h_1^{3/2} - h_2^{3/2} \right]
\]

\[
= \frac{5A}{4 C_d \sqrt{2g \tan(\theta/2)}} \left[ \frac{1}{H_2^{1/2}} - \frac{1}{H_1^{1/2}} \right] = \frac{5 \times 375}{4 \times 0.6 \sqrt{2} \times 9.81 (1)} \left[ \frac{1}{\sqrt{1.5^3}} - \frac{1}{\sqrt{1.5^3}} \right] = 390 \text{ sec} = 6 \text{ min}, 30 \text{ sec}
\]
Exercise

6.1 A weir 8 m length is to be built across a rectangular channel to discharge a flow of $9 \text{ m}^3/\text{s}$. If the maximum depth of water on the upstream side of weir is to be 2 m, what should be the height of the weir? $C_d = 0.62$.

6.2 A rectangular notch 1 m long and 40 cm high is discharging water. If the same quantity of water be allowed to flow over a 90° V-notch, find the height to which water will rise above the apex of notch. $C_d = 0.62$.

6.3 A Venturi meter with a 15 cm I.D. at inlet and 10 cm I.D. at throat is laid with its axis horizontal and is used for measuring the flow of oil of sp.gr. = 0.9. The oil-mercury differential manometer shows a gauge difference of 20 cm. If $C_d = 0.98$, calculate the discharge of oil.

6.4 Find the throat diameter of a Venturi meter when fitted to a horizontal pipe 10 cm diameter having a discharge of 20lit/s. The differential U-tube mercury manometer, shows a deflection giving a reading of 60 cm, $C_d = 0.98$. In case, this Venturi meter is introduced in a vertical pipe, with the water flowing upwards, find the difference in the reading of mercury gauge. The dimensions of pipe and Venturi meter remain unaltered, as well as the discharge through the pipe.
MODULE VII

LIQUID MIXING

7.1. Introduction

Mixing is one of the most common operations carried out in the chemical, processing and allied industries. The term "mixing" is applied to the processes used to reduce the degree of non-uniformity, or gradient of a property in a system such as concentration, viscosity, temperature and so on. Mixing is achieved by moving material from one region to another. It may be of interest simply as a means of achieving a desired degree of homogeneity but it may also be used to promote heat and mass transfer, often where a system is undergoing a chemical reaction. Another closely related operation is Agitation which is not synonymous to mixing. Agitation refers to the induced motion of a material in a specified way, usually in a circulatory pattern inside some sort of container while mixing is the random distribution, into and through one another of two or more initially separate phases. A single homogeneous material such as a tank full of cold water can be agitated but it cannot be mixed until some other material such as a quantity of hot water or some powdered solid is added to it.

A mixing equipment which may be a rotating agitator generates high velocity streams of liquid, which in turn entrain stagnant or slower moving regions of liquid resulting in uniform mixing by momentum transfer. This equipment may then be designed not only to achieve a predetermined level of homogeneity, but also to improve heat transfer. For example, the rotational speed of an impeller in a mixing vessel is selected so as to achieve a required rate of heat transfer, and the agitation may then be more than sufficient for the mixing duty. A typical example of this equipment is a shown in Figure 7.1

Several attempts have been made to classify mixing problems and prominent classification usually adopted has been that which describes. This is probably the most useful description of mixing as it allows the adoption of a unified approach to the problems encountered in a range of industries. This includes:

**Single-phase liquid mixing**: Blending miscible liquids to give a product of a desired specification, such as, in the blending of petroleum products of different viscosities.

**Mixing of immiscible liquids**: dispersing a second immiscible liquid with the first to form an emulsion of suspension of fine drops. Liquid-liquid extraction is one important example of this type of mixing.
**Gas-liquid mixing**: dispersing a gas through the liquid in the form of small bubbles. Examples are aerobic fermentation, wastewater treatment, oxidation of hydrocarbons. The purpose of mixing here is to produce a high interfacial area. Generally, gas liquid mixtures or dispersions are unstable and separate rapidly if agitation is stopped, provided that a foam is not formed when a surface-active agent is added.

**Liquid-solids mixing**: Mechanical agitation may be used to suspend particles in a liquid in order to promote mass transfer or a chemical reaction. The liquids involved in such applications are usually of low viscosity, and the particles will settle out when agitation ceases. At the other extreme, in the formation of composite materials, especially filled polymers, fine particles must be dispersed into a highly viscous Newtonian or non-Newtonian liquid. The incorporation of carbon black powder into rubber is one example.

**Gas-liquid-solids mixing**: In some applications such as catalytic hydrogenation of vegetable oils, slurry reactors, froth flotation, evaporative crystallisation, and so on, the success and efficiency of the process is directly influenced by the extent of mixing between the three phases.

**Solids-solids mixing**: Mixing together of particulate solids, sometimes referred to as blending, is a very complex process in that it is very dependent, not only on the character of the particles — density, size, size distribution, shape and surface properties — but also on the differences of these properties in the components. Mixing of sand, cement and aggregate to form concrete and of the ingredients in gunpowder preparation are longstanding examples of the mixing of solids. Other industrial sectors employing solids mixing include food, drugs, and the glass industries. All these applications involve only physical contacting.

### 7.2 Mixing Mechanisms

If mixing is to be carried out in order to produce a uniform mixture, it is necessary to understand how liquids move and approach this condition. In liquid mixing devices, it is necessary that two requirements are fulfilled. Firstly, there must be bulk or convective flow so that there are no dead (stagnant) zones. Secondly, there must be a zone of intensive or high-shear mixing in which the non-homogeneities are broken down. All these factors are important in mixing, which can be described as a combination of three physical processes: **distribution, dispersion and diffusion**. These processes are energy-consuming and ultimately the mechanical energy is dissipated as heat; the proportion of energy attributable to each varies from one application to another. Depending upon the fluid properties, primarily viscosity, the flow in mixing vessels may be laminar or turbulent, with a substantial transition zone in between the two, and frequently both flow types will occur simultaneously in different parts of the vessel. Laminar and turbulent flow arise from different mechanisms, and it is convenient to consider them separately.

#### 7.2.1 Laminar Mixing
Laminar flow is usually associated with high viscosity liquids (in excess of 10 N s/m²) which may be either Newtonian or non-Newtonian. This makes the inertial forces to die out quickly, and the impeller of the mixer must cover a significant proportion of the cross-section of the vessel to impart sufficient bulk motion. Here, the velocity gradients close to the rotating impeller are high, the fluid elements in that region deform and stretch (thinning of fluid elements) through (i) laminar shear flow (ii) extensional or elongational flow and (iii) molecular diffusion.

Both mechanisms of shear (Figure 7.2a) and elongation (Figure 7.2b), give rise to stresses in the liquid which then effect a reduction in droplet size and an increase in interfacial area, by which means the desired degree of homogeneity is obtained. In addition, molecular diffusion is always tending to reduce inhomogeneities but its effect is not significant until the fluid elements have been reduced in size sufficiently for their specific areas to become large. It must be recognized, however, that the ultimate homogenisation of miscible liquids, can be only brought about by molecular diffusion. In the case of liquids of high viscosity, this is a slow process.

For low viscosity liquids (less than 10 mN s/m²), the bulk flow pattern in mixing vessels with rotating impellers is turbulent. The inertia imparted to the liquid by the rotating impeller is sufficient to cause the liquid to circulate throughout the vessel and return to the impeller. Turbulent eddy diffusion takes place throughout the vessel but is a maximum in the vicinity of the impeller. Eddy diffusion is inherently much faster than molecular diffusion and, consequently, turbulent mixing occurs much more rapidly than laminar mixing. Ultimately homogenisation at the molecular level depends on molecular diffusion, which takes place more rapidly in low viscosity liquids. Mixing is most rapid in the region of the impeller because of the high shear rates due to the presence of trailing vortices, generated by disc-turbine impellers, and associated Reynolds stresses. Furthermore, a high proportion of the energy is dissipated here.

Turbulent flow is inherently complex, and calculation of the flow fields prevailing in a mixing vessel is not amenable to rigorous theoretical treatment. If the Reynolds numbers of the main flow is sufficiently high, some insight into the mixing process can be gained by using the theory of local isotropic turbulence

7.3. Mixing Equipment
The wide range of mixing equipment available commercially reflects the enormous variety of mixing duties encountered in the processing industries. It is reasonable to expect therefore that no single item of mixing equipment will be able to carry out such a range of duties effectively. This has led to the development of a number of distinct types of mixer over the years. The choice of a mixer type and its design is primarily governed by experience. In the following sections, the main mechanical features of commonly used types of equipment together with their range of applications are described.

7.3.1. Mechanical agitation (Agitator)
This is perhaps the most commonly used method of mixing liquids (Figure 7.1), and essentially there are three elements in such devices.

7.3.1.1 Vessels
These are often vertically mounted cylindrical tanks, up to 10 m in diameter, which typically are filled to a depth equal to about one diameter, although in some gas-liquid contacting systems tall vessels are used and the liquid depth is up to about three tank diameters; multiple impellers fitted on a single shaft are then frequently used. The base of the tanks may be flat, dished, or conical, or specially contoured, depending upon factors such as ease of emptying, or the need to suspend solids. Other horizontally mounted vessels are also available such as those for the batch mixing of viscous pastes and doughs using ribbon impellers and Z blade mixers.

7.3.1.2 Baffles
To prevent gross vortexing, which is detrimental to mixing, particularly in low viscosity systems, baffles are often fitted to the walls of the vessel. These take the form of thin strips about one-tenth of the tank diameter in width, and typically four equi-spaced baffles may be used. In some cases, the baffles are mounted flush with the wall, although occasionally a small clearance is left between the wall and the baffle to facilitate fluid motion in the wall region. Baffles are, however, generally not required for high viscosity liquids because the viscous shear is then sufficiently great to damp out the rotary motion. Sometimes, the problem of vortexing is circumvented by mounting impellers off-centre.

7.3.1.3 Impellers
Some of the impellers which are frequently used are as shown in Figure 7.3. Propellers, turbines, paddles, anchors, helical ribbons and screws are usually mounted on a central vertical shaft in a cylindrical tank, and they are selected for a particular duty largely on the basis of liquid viscosity. Propellers, turbines and paddles are generally used with relatively low viscosity systems and operate at high rotational speeds. A typical velocity for the tip of the blades of a turbine is of the order of 3 m/s, with a propeller being a little faster and the paddle a little slower. These are classed as remote-clearance impellers, having diameters in the range (0.13 - 0.67) x (tank diameter). Anchors, helical ribbons and screws, are generally used for high viscosity liquids. The anchor and ribbon are
arranged with a close clearance at the vessel wall, whereas the helical screw has a smaller diameter and is often used inside a draft tube to promote fluid motion throughout the vessel. Helical ribbons or interrupted ribbons are often used in horizontally mounted cylindrical vessels. A guide for the selection of a type of impeller is given in Table 7.1.

![Figure 7.3: Commonly used impellers](image)

**Figure 7.3: Commonly used impellers (a) Three-bladed propeller (b) Six-bladed disc turbine (Rushton turbine) (c) Simple paddle (d) Anchor impeller (e) Helical ribbon (/) Helical screw with draft tube (g) Z-blade mixer (h) Banbury mixer**

**Table 7.1: Impeller selection guide**

<table>
<thead>
<tr>
<th>Type of impeller</th>
<th>Range of liquid, cP</th>
<th>Viscosity, kg/m - sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anchor</td>
<td>$10^2 - 2 \times 10^3$</td>
<td>$10^{-1} - 2$</td>
</tr>
<tr>
<td>Propeller</td>
<td>$10^0 - 10^4$</td>
<td>$10^{-3} - 10^1$</td>
</tr>
<tr>
<td>Flat-blade turbine</td>
<td>$10^0 - 3 \times 10^4$</td>
<td>$10^{-3} - 3 \times 10^1$</td>
</tr>
<tr>
<td>Paddle</td>
<td>$10^2 - 3 \times 10^1$</td>
<td>$10^{-1} - 3 \times 10^1$</td>
</tr>
<tr>
<td>Gate</td>
<td>$10^3 - 10^5$</td>
<td>$10^0 - 10^2$</td>
</tr>
<tr>
<td>Helical screw</td>
<td>$3 \times 10^3 - 3 \times 10^5$</td>
<td>$3 - 3 \times 10^2$</td>
</tr>
<tr>
<td>Helical ribbon</td>
<td>$10^4 - 2 \times 10^6$</td>
<td>$10^1 - 2 \times 10^3$</td>
</tr>
<tr>
<td>Extruders</td>
<td>$&gt;10^5$</td>
<td>$&gt;10^3$</td>
</tr>
</tbody>
</table>


7.3.2. Portable mixers

For a wide range of applications, a portable mixer which can be clamped on the top or side of the vessel is often used. This is commonly fitted with two propeller blades so that the bottom rotor...
forces the liquid upwards and the top rotor forces the liquid downwards. The power supplied is up to about 2 kW, though the size of the motor becomes too great at higher powers. To avoid excessive strain on the armature, some form of flexible coupling should be fitted between the motor and the main propeller shaft. Units of this kind are usually driven at a fairly high rate (15 Hz), and a reduction gear can be fitted to the unit fairly easily for low-speed operations although this increases the mass of the unit.

7.7.3. Extruders
Mixing duties in the plastics industry are often carried out in either single or twin screw extruders. The feed to such units usually contains the base polymer in either granular or powder form, together with additives such as stabilisers, pigments, plasticisers, and so on. During processing in the extruder the polymer is melted and the additives mixed. The extrudate is delivered at high pressure and at a controlled rate from the extruder for shaping by means of either a die or a mould. In the typical single-screw shown in Figure 7.4a, the shearing which occurs in the helical channel between the barrel and the screw is not intense, and therefore this device does not give good mixing. Twin screw extruders, as shown in Figure 7.4b, may be co or counter-rotating, and here, there are regions where the rotors are in close proximity thereby generating extremely high shear stresses. Clearly, twin-screw units can yield a product of better mixture quality than a single-screw machine.

7.3.4. Static mixers
All the mixers described so far have been of the dynamic type in the sense that moving blades are used to impart motion to the fluid and produce the mixing effect. In static mixers, sometimes called "in-line" or "motionless" mixers, the fluids to be mixed are pumped through a pipe containing a series of specially shaped stationary blades. Static mixers can be used with liquids of a wide range of viscosities in either the laminar or turbulent regimes, but their special features are perhaps best appreciated in relation to laminar flow mixing. The flow patterns within the mixer are complex.

A particular type of static mixer in which a series of stationary helical blades mounted in a circular pipe is used to divide and twist the flowing streams is as shown on Figure 7.5a. In laminar flow, (Figure 7.5b) the material divides at the leading edge of each of these elements and follows the channels created by the element shape.
Figure 7.5a: Twisted-blade type of static mixer elements

Figure 7.5b: Twisted-blade type of static mixer operating in the laminar flow regime (a) Distributive mixing mechanism showing, in principle, the reduction in striation thickness produced (h) Radial mixing contribution from laminar shear mechanism

7.4. Scale-Up of Stirred Vessels

One of the problems confronting the designers of mixing equipment is that of deducing the most satisfactory arrangement for a large unit from experiments with small units. In order to achieve the same kind of flow pattern in two units, geometrical, kinematic, and dynamic similarity and identical boundary conditions must be maintained. It has been found convenient to relate the power used by the agitator to the geometrical and mechanical arrangement of the mixer, and thus to obtain a direct indication of the change in power arising from alteration of any of the factors relating to the mixer. A typical mixer arrangement is shown in Figure 7.6.

For similarity in two mixing systems, it is important to achieve geometric kinematic and dynamic similarity.

**Geometric similarity**: prevail between two systems of different sizes if all counterpart length dimensions have a constant ratio. Thus the following ratios must be the same in two systems:
Kinematic similarity exists in two geometrically similar units when the velocities at corresponding points have a constant ratio. Also, the paths of fluid motion (flow patterns) must be alike.

Dynamic similarity occurs in two geometrically similar units of different sizes if all corresponding forces at counterpart locations have a constant ratio. It is necessary here to distinguish between the various types of force: inertial, gravitational, viscous, surface tension and other forms, such as normal stresses in the case of viscoelastic non-Newtonian liquids. Some or all of these forms may be significant in a mixing vessel. Considering corresponding positions in systems 1 and 2 which refer to the laboratory and large scale, respectively, when the different types of force occurring are $F_a$, $F_b$, $F_c$ and so on, dynamic similarity requires that:

$\frac{F_{a1}}{F_{a2}} = \frac{F_{b1}}{F_{b2}} = \frac{F_{c1}}{F_{c2}} = \ldots$ a constant

$\frac{F_{a1}}{F_{b1}} = \frac{F_{a2}}{F_{b2}} = \frac{F_{a3}}{F_{b3}} = \ldots$ euc.

Some of the various types of forces that may arise during mixing or agitation will be formulated:

1- **Inertial Force** [$F_i$]

Is associated with the reluctance of a body to change its state of rest or motion.

The inertial force ($F_i$) $= \text{(mass)} \times \text{(acceleration)} = m \cdot a$

$\text{d}F_i = \text{d}m \cdot (\text{d}u/\text{d}t)$

but $m = \rho \cdot V = \rho \cdot A \cdot L$

$\Rightarrow \text{d}m = \rho \cdot \text{d}V = \rho \cdot A \cdot \text{d}L$

and $u = \text{d}L/\text{d}t$

$\Rightarrow \text{d}F_i = \rho \cdot A \cdot \text{d}L \cdot \text{d}u/\text{d}t = \rho \cdot A \cdot (\text{d}L/\text{d}t) \cdot \text{d}u = \rho \cdot A \cdot u \cdot \text{d}u$

$\Rightarrow F_i = \int_0^{\text{d}F_i} = \int_0^\text{d}u \cdot \rho \cdot A \cdot u \cdot \text{d}u = \rho \cdot A \cdot u^2/2$

In mixing applications;

$A \propto D_A^{-2}$

$D_A$: diameter of agitator

$u = \pi \cdot D_A \cdot N$

$N$: rotational speed

Therefore, the expression for inertial force may be written as;

$F_i \propto \rho \cdot D_A^{-2} \cdot N^2$
2- **Viscous Force** \( F_v \)

The viscous force for Newtonian fluid is given by:

\[
F_v = \mu A \frac{du}{dy}
\]

In mixing applications;

\[
A \propto D_\lambda^2; \quad \frac{du}{dy} \propto \pi D_\lambda N / D_\lambda
\]

Therefore, the expression for viscous force may be written as:

\[
F_v \propto \mu D_\lambda^2 N
\]

3- **Gravity Force** \( F_g \)

The inertial force \( F_g \) = (mass) (gravitational acceleration) = m.g

In mixing applications;

\[
m = \rho V = \rho A L \propto \rho D_\lambda^3
\]

\[
F_g \propto \rho D_\lambda^3 g
\]

4- **Surface Tension Force** \( F_s \)

In mixing applications;

\[
F_s \propto \sigma D_\lambda
\]

In the design of liquid mixing systems the following dimensionless groups are of importance:

1- **The Power Number** (\( N_p \))

\[
N_p = \frac{P_A}{\rho N^4 D_\lambda^5}
\]

where, \( P_A \): is the power consumption.

2- **The Reynolds Number** (\( Re_m \))

\[
(Re)_m = \frac{\text{Inertial Force}}{\text{Viscous Force}} = \frac{F_i}{F_v} = \frac{\rho D_\lambda^4 N^2}{\mu D_\lambda^2 N}
\]

\[
\Rightarrow (Re)_m = \frac{\rho N D_\lambda^2}{\mu}
\]

3- **The Froude Number** (\( Fr_m \))

This number related to fluid surface [related to vortex system in mixing]

\[
(Fr)_m = \frac{\text{Inertial Force}}{\text{Gravity Force}} = \frac{F_i}{F_g} = \frac{\rho D_\lambda^4 N^2}{\rho D_\lambda^4 g}
\]

\[
\Rightarrow (Fr)_m = \frac{N^2 D_\lambda}{g}
\]

4- **The Weber Number** (\( We_m \))

This number related to multiphase fluids [or fluid flow with interfacial forces]

\[
(We)_m = \frac{\text{Inertial Force}}{\text{Surface Tension Force}} = \frac{F_i}{F_s} = \frac{\rho D_\lambda^4 N^2}{\sigma D_\lambda}
\]

\[
\Rightarrow (We)_m = \frac{\rho N^2 D_\lambda^3}{\sigma}
\]
It can be shown by **dimensional analysis** that the power number (Np) can be related to the Reynolds number (Re)_m and the Froude number (Fr)_m by the equation;

\[ Np = C(Re)_m^a (Fr)_m^b \]

where, C is an overall dimensionless shape factor which represents the geometry of the system.

The last equation can also be written in the form;

\[ \Phi = \frac{Np}{(Fr)_m} = C(Re)_m^a \]

where, \( \Phi \) is defined as the dimensionless power function.

**The Froude number (Fr)_m** is usually important only in situations where gross vortexting. Since vortexting is a gravitational effect, the (Fr)_m is not required to describe a baffled liquid mixing systems. In this case the exponent of (Fr)_m (i.e. y) in the last two equations is zero. [ (Fr)_m^y = (Fr)_m^1 = 1 \Rightarrow \Phi = Np].

Thus the non-vortexting systems, the equation of power function (\( \Phi \)) can be written wither as;

\[ \Phi = Np = C(Re)_m^a \text{ or as; } \log C = \log Np = \log C + x \log(Re)_m \]

The Weber number of mixing (We)_m is only of importance when separate physical phases are present in the liquid mixing system as in liquid-liquid extraction.

**7.5 Power Curve**

A power curve is a plot of the power function (\( \Phi \)) or the power number (Np) against the Reynolds number of mixing (Re)_m on log-log coordinates. Each geometrical configuration has its own power curve and since the plot involves dimensionless groups it is independent of tank size. Thus a power curve that used to correlate power data in a 1 m^3 tank system is also valid for a 1000 m^3 tank system provided that both tank systems have the same geometrical configuration.

The Figure below shows the power curve for the standard tank configuration. Since this is a baffled tank (non-vortexting system), the following equation is applied;

\[ \log \Phi = \log Np = \log C + x \log(Re)_m \]  \[ (22) \]

From the Figure it is clear that the power curve for the standard tank configuration is linear in the laminar flow region (line-AB) with slope (-1) in this region [(Re)_m < 10]. Then the last equation can be written in the following form;

\[ \log \Phi = \log Np = \log C + \log(Re)_m \]

which can be rearranged to give;

\[ P_\lambda = C \mu N^2 D_\lambda^3 \]

C is a constant depend on the type of agitator and vessel arrangement and if the tank is with or without baffles. For the standard tank configuration \( C = 71 \) and for marine type 3-blade \( C = 41 \). Thus for the laminar flow, power (\( P_\lambda \)) is directly proportional to dynamic viscosity (\( \mu \)) for a fixed agitator speed (N).
Figure 7.6: Power Curve for the Standard Tank Configuration with Baffles

For the transition flow region BCD which extends up to \((Re)_m = 10,000\), the constant \(C\) and the slope \(x\) in equation \(\Phi\) vary continuously.

In fully turbulent flow \((Re)_m > 10,000\), the curve becomes horizontal and the power function \(\Phi\) is independent of Reynolds number of mixing \((Re)_m\) i.e.
\[
\Phi = Np = 6.3\quad \text{for } (Re)_m > 10,000
\]

At point (C) on the power curve, for the standard tank configuration, enough energy is being transferred to the liquid for vortexing to start. However the baffles in the tank prevent this.

If the baffles were not present, vortexing would develop and the power curve would be as shown in Figure below;

Figure 7.7: Power Curve for the Standard Tank Configuration without Baffles

The power curve for the baffled system is identical with that for the unbaffled system up to point (C) where \([(Re)_m \approx 300]\). As the \((Re)_m\) increases beyond point (C) in the unbaffled system, vortexing increases and the power falls sharply as shown in the above Figure 7.7
Example 7.1
Calculate the theoretical power in Watt for a 3 m diameter, 6-blade flat blade turbine agitator running at 0.2 rev/s in a tank system conforming to the standard tank configuration. The liquid in the tank has a dynamic viscosity of 1 Pa.s, and a liquid density of 1000 kg/m³

Solution:

\[(Re)_m = \frac{\rho N D_A^2}{\mu} = \frac{(1000)(0.2)(3)^2}{1} = 1,800\]

From Figure (1) \(\Phi = Np = 4.5\)

The theoretical power for mixing

\[P_A = \frac{Np \rho N^3 D_A^2}{\beta} = \frac{4.5(1000)(0.2)^3(3)^2}{10} = 8,748 \text{ W}\]

Example 7.2:
Calculate the theoretical power in Watt for a 0.1 m diameter, 6-blade flat blade turbine agitator running at 16 rev/s in a tank system without baffles and conforming to the standard tank configuration. The liquid in the tank has a dynamic viscosity of 0.08 Pa.s, and a liquid density of 900 kg/m³

Solution:

\[(Re)_m = \frac{\rho N D_A^2}{\mu} = \frac{(900)(16)(0.1)^2}{0.08} = 1,800\]

From Figure (2) \(\Phi = 2.2\)

The theoretical power for mixing

\[P_A = \frac{\Phi (Fr)_m^\gamma \rho N^3 D_A^2}{\beta} \Rightarrow y = \frac{1 - \log(1800)}{40} = 0.05638\]

\[(Fr)_m = \frac{N^2 D_A}{g} = \frac{(16)(0.1)^2}{9.81} = 2.61\]

\[\Rightarrow P_A = 2.2 \times (0.9479)(900)(16)^2(0.1) \approx 76.88 \text{ W}\]